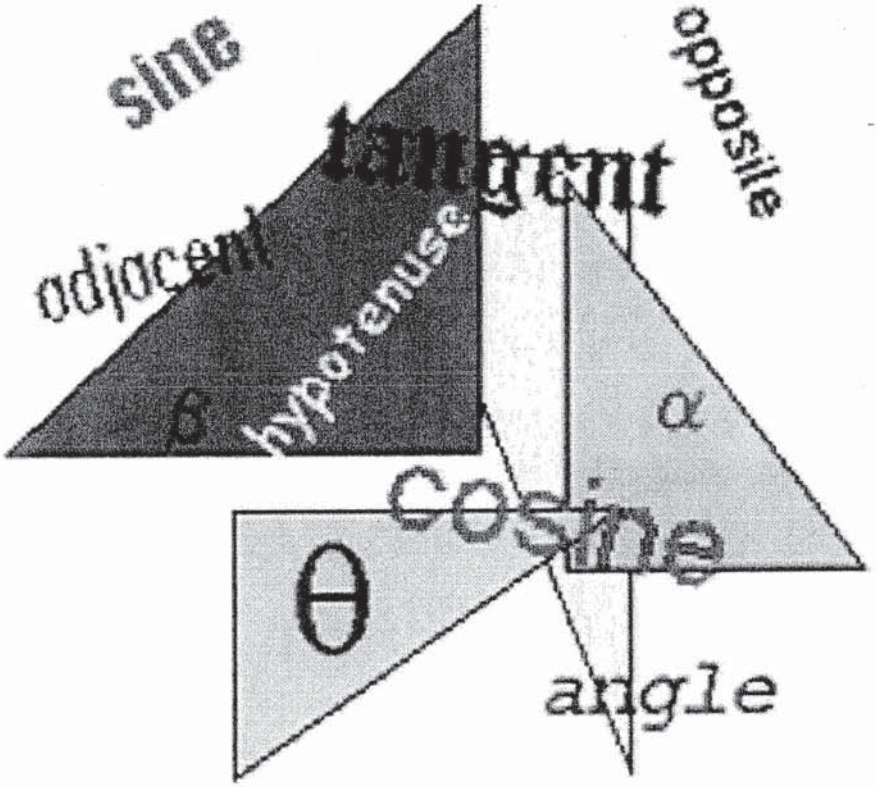
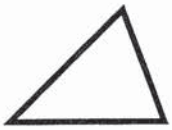
Trigonometry



Triangle Review

A _____ triangle has no sides and no angles equal.

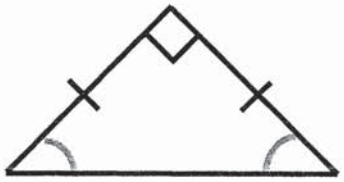


An _____ triangle has two sides and two angles equal.

An _____ triangle has three sides and three angles equal.

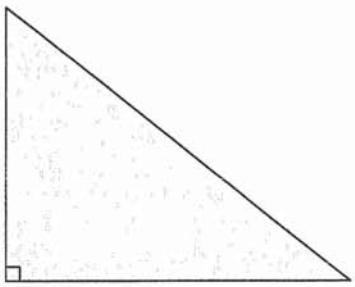
A _____ triangle has one right angle.

Identify the triangle below;

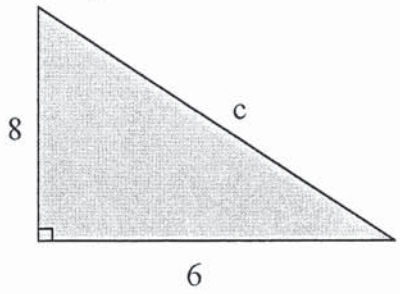


How are the three sides of a right triangle related to each other?

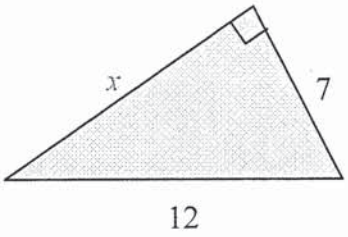
The Pythagorean Theorem



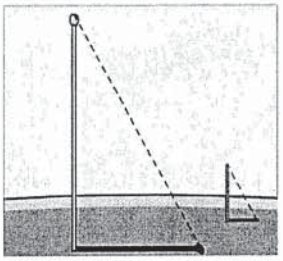
Example 1: Calculate side c.



Example 2: Calculate side *x*.



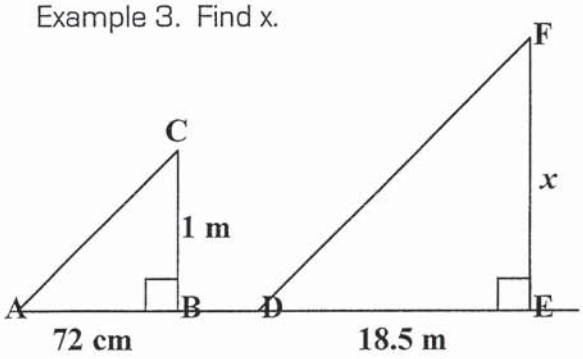
Similar Triangles



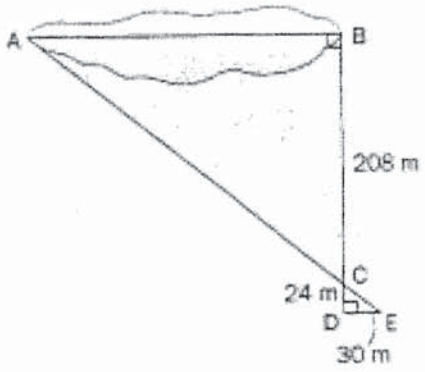
Two triangles are considered to be similar if and only if:

- they have the same shape
- corresponding angles are equal
- the ratio of the corresponding side lengths are equal

Example 3. Find x.



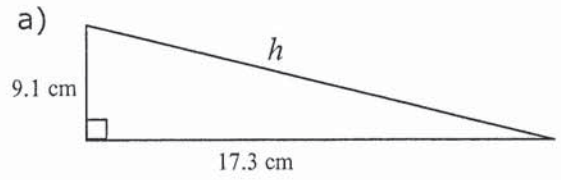
Example 4. Surveyors have laid out triangles to find the length of a lake. Calculate this length, AB.



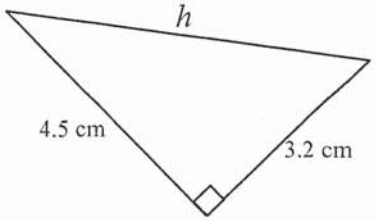
Review: Pythagorean Theorem and Similar Triangles

1. In each triangle, determine the length of the hypotenuse. (19.55, 5.52)

a)

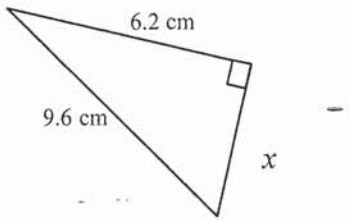


b)

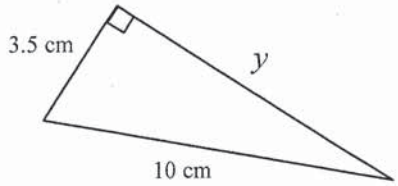


2. Determine each length. (7.33, 9.37)

a)



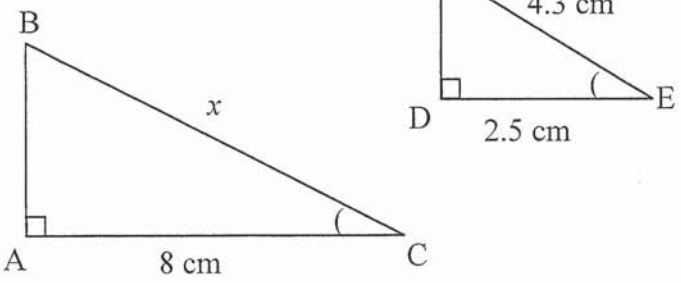
b)



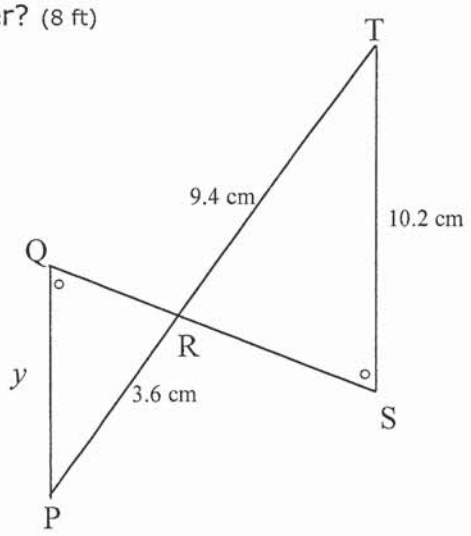
3. A 10 ft ladder is leaning on a 6 ft wall. If the top of the ladder just touches the top of the wall, how far from the base of the wall is the ladder? (8 ft)

4. Determine each length. (13.76, 3.91)

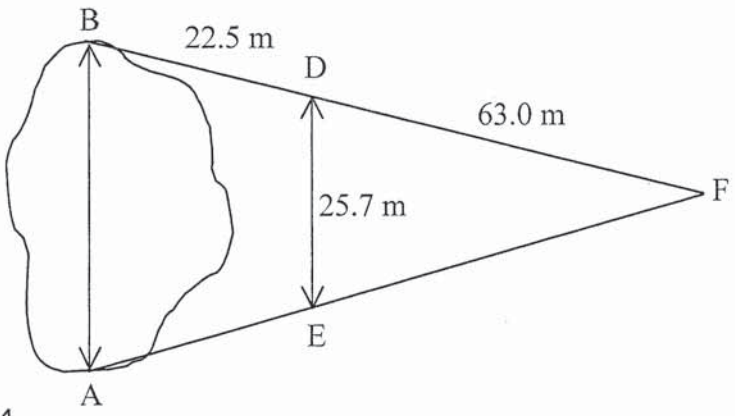
a)



b)



5. To find the distance AB across a pond, surveyors measured the distances shown. Use these distances to calculate the distance AB. (34.88 m)



Trigonometry

The word trigonometry means "triangle measurement." You can use trigonometry to calculate the lengths of sides and the measures of angles in triangles.

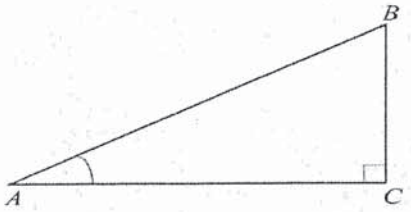
Trigonometry has been used for centuries in the study of:

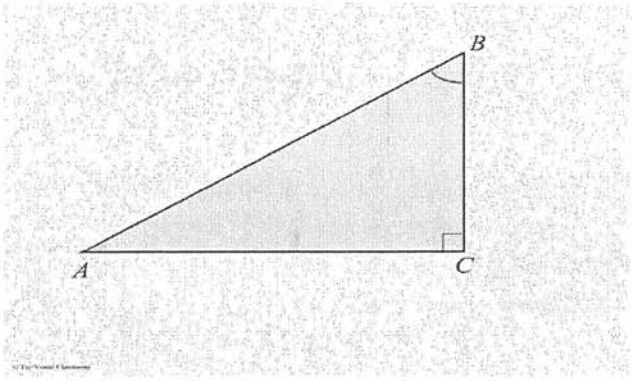
surveying

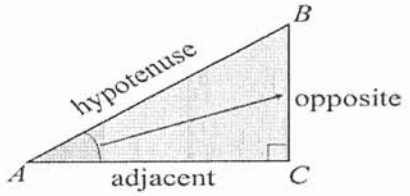
astronomy **geography**

engineering **physics**

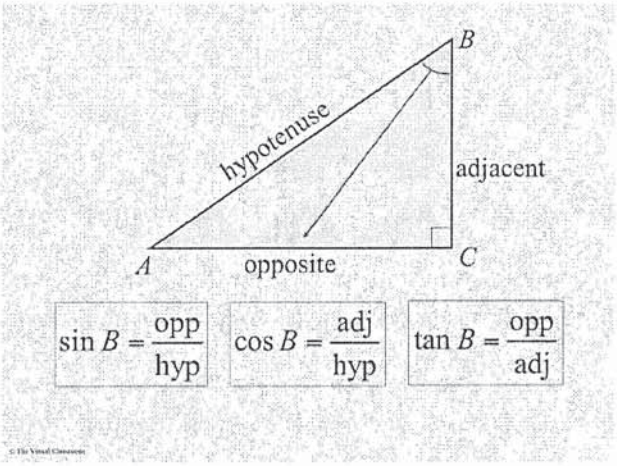
Parts of a Right Triangle



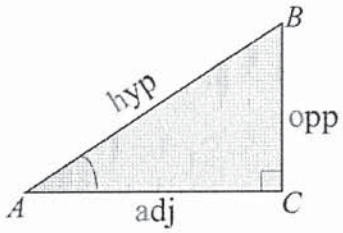




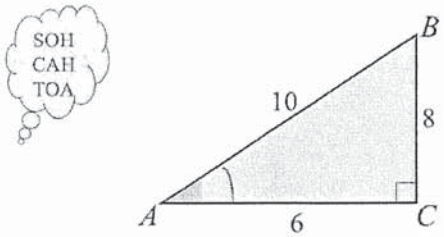
$\sin A = \frac{\text{opp}}{\text{hyp}}$ $\cos A = \frac{\text{adj}}{\text{hyp}}$ $\tan A = \frac{\text{opp}}{\text{adj}}$

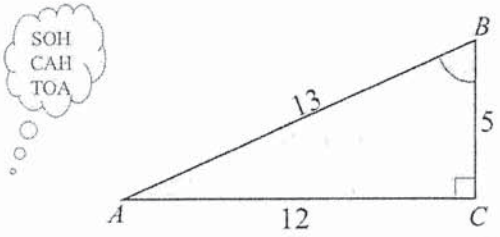


$\sin B = \frac{\text{opp}}{\text{hyp}}$ $\cos B = \frac{\text{adj}}{\text{hyp}}$ $\tan B = \frac{\text{opp}}{\text{adj}}$



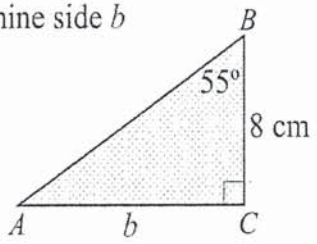
SOH CAH TOA



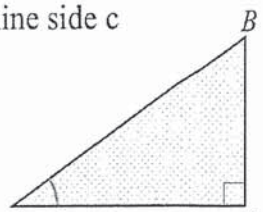


Example 1: Determine side b

SOH
CAH
TOA

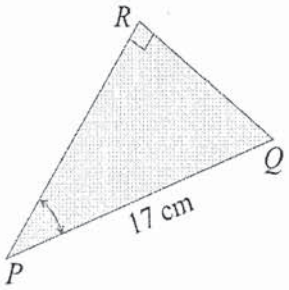


Example 2: Determine side c if $\angle A$ is 31°.



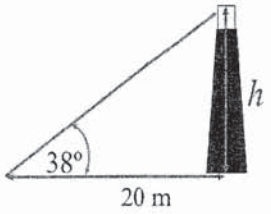
SOH
CAH
TOA

Example 3: Determine side p if $\angle P$ is 45°.



SOH
CAH
TOA

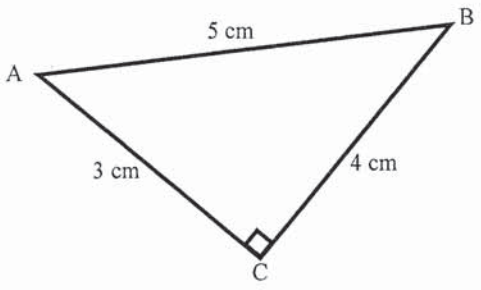
Ex. 4: From a distance of 20 m from the base a lighthouse the angle of elevation to the top of a lighthouse is 38°. Determine the height of the lighthouse.

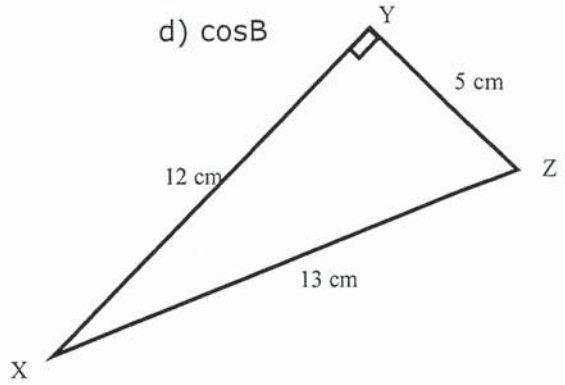


Trig Ratios: Finding the Length
Worksheet

1. For DABC below left, write the ratio of sides for each trigonometric ratio.

a) sinA b) cosA c) tanB d) cosB





2. For $\angle X$ in ΔXYZ above right, identify the trigonometric ratio equal to each fraction.

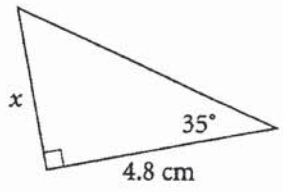
a) $\frac{5}{12}$ b) $\frac{5}{13}$ c) $\frac{12}{13}$

3. Determine each trigonometric ratio to 4 decimals places.

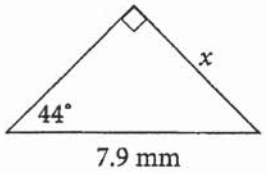
a) cos49° b) tan81° c) sin52° d) tan22°
e) sin17° f) cos63° g) tan9° h) sin66°

4. Determine length x in each triangle. Give the length to 1 decimal place.

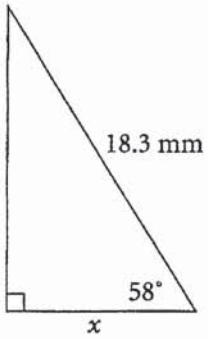
a)



b)

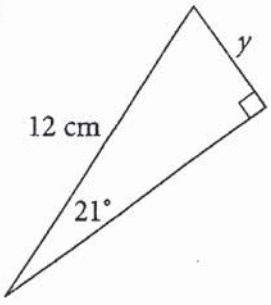


c)

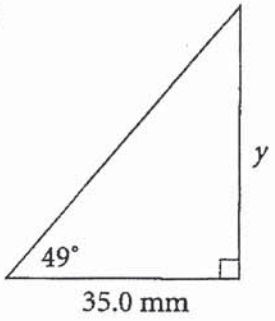


5. Determine length y in each triangle. Give the length to the nearest unit.

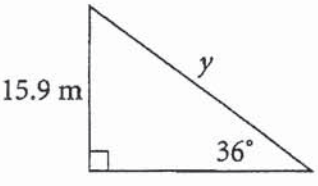
a)



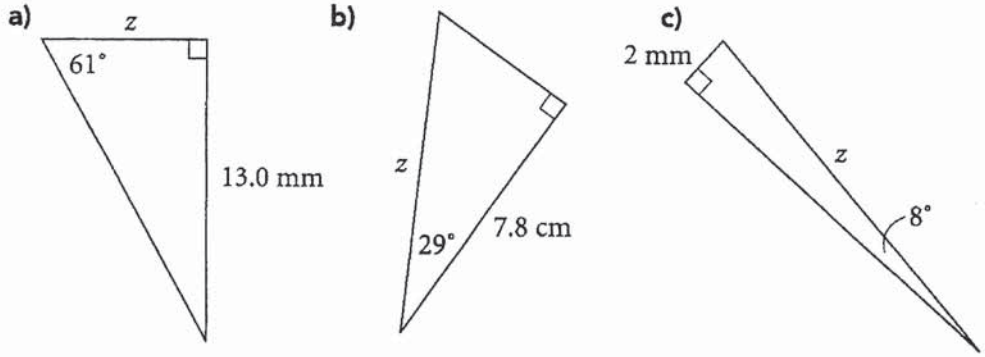
b)



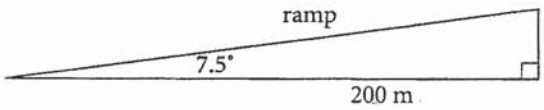
c)



6. Determine length z in each triangle. Give the length to the nearest tenth of a unit.



7. A ramp has an incline of 7.5°. The horizontal length of the ramp is 200 m.



a) To the nearest metre, what is the vertical height of the ramp?

b) Calculate the length of the ramp. Give the length to the nearest metre.

8. For each triangle, determine the length indicated. Answer to the nearest unit.

a) ΔABC: $\angle C = 47°$, $\angle B = 90°$, and $a = 8$ cm; calculate c.

b) ΔXYZ: $\angle Y = 90°$, $\angle Z = 32°$, and $y = 15$ m; calculate x.

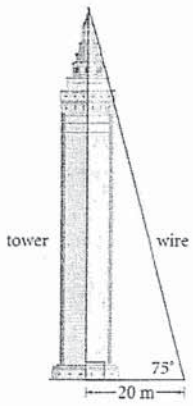
c) ΔLMN: $\angle M = 90°$, $\angle N = 64°$, and, $m = 12$ mm; calculate n.

9. For each triangle, determine the length indicated. Answer to the nearest unit.

a) ΔXYZ: $\angle X = 41°$, $\angle Y = 90°$, and $x = 11.3$ m; calculate y.

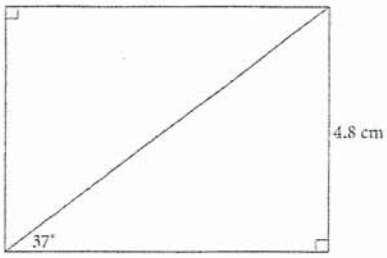
b) ΔABC: $\angle A = 90°$, $\angle C = 38°$, and $b = 9.4$ cm; calculate c.

c) ΔDEF: $\angle E = 90°$, $\angle F = 63°$, and, $d = 4.5$ mm; calculate e.



10. A guy wire is attached to the top of a vertical tower and to an anchor point on horizontal ground 20 m from the base of a tower. The guy wire makes an angle of 75° with the ground. Determine the height of the tower to the nearest metre.

11. A rectangle is 4.8 cm wide (above right). A diagonal of the rectangle makes an angle 37° with the longer side of the rectangle.

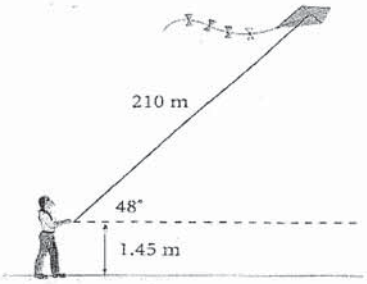


a) Determine the length of the diagonal to the nearest millimetre.

b) Determine the length of the rectangle to the nearest millimetre.

12. A ladder is 3.0 m long. It is leaning against a wall. The foot of the ladder makes an angle of 76° with the ground. How far up the wall is the top of the ladder? Give the length to the nearest tenth of a metre.

13. Sam is flying a kite. She has let out the string to a length of 210 m. The string makes an angle of 48° with the horizontal. The string is taut and Sam's hand is 1.45 m above the ground.



    a) What is the vertical distance between Sam's hand and the kite?

    b) How high is the kite above the ground?

14. Triangle ABC has these measurements: $\angle B = 90°$, $\angle A = 34°$, and $b = 14$ cm.

    a) Sketch ΔABC.

    b) Which of the two legs do you think is longer? Explain your answer.

    c) Determine the lengths a and c to verify your answer in part b.

15. In ΔXYZ, $\angle Y = 90°$, $\angle Z = 45°$, and $y = 8.6$ cm.

    a) What should be true about the lengths of the two legs? Explain.

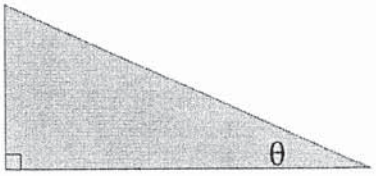
    b) Calculate to verify your answer in part a.

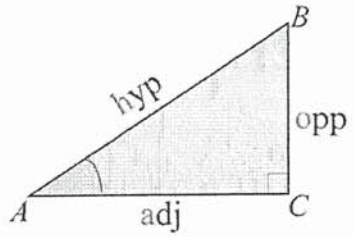
    c) What statements can you make about sin45° and cos45°? Explain.

Answers

1. a) 4/5 b) 3/5 c) 3/4 d) 4/5 2. a) tanX b) sinX c) cosX 3 a) 0.6561 b) 6.3138 c) 0.7880 d) 0.4040 e) 0.2924 f) 0.4540 g) 0.1584 h) 0.9135 4. a) 3.36 cm b) 5.49 mm c) 9.70 mm 5. a) 4.3 cm b) 40.26 mm c) 27.05 m 6. a) 7.2 mm b) 8.9 cm c) 14.4 mm 7. a) 26.3 m b) 201.7 m 8. a) 8.58 cm b) 12.7 m c) 10.79 mm 9. a) 17.22 m b) 7.34 cm c) 9.9 mm 10. 74 m 11. a) 7.98 cm b) 6.37 cm 12. 2.91 m 13. a) 156.06 m b) 157.51 m 14. b) c is longer because C is the larger angle c) a = 7.83 cm, c = 11.6 cm 15. a) x and z are equal because X and Z are equal (isosceles) b) x = 6.08 cm, z = 6.08 cm c) sin45 = cos45

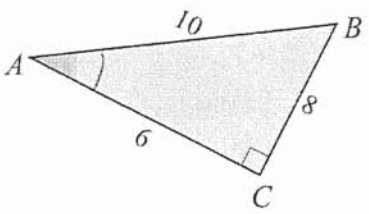
Trig Ratios - Finding Angles



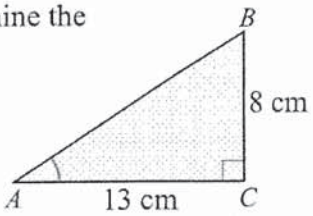


$$\sin A = \frac{\text{opp}}{\text{hyp}} \qquad \cos A = \frac{\text{adj}}{\text{hyp}} \qquad \tan A = \frac{\text{opp}}{\text{adj}}$$

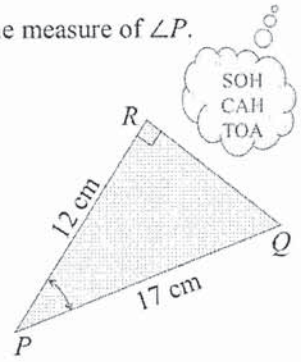
$$A = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) \qquad A = \cos^{-1}\left(\frac{\text{adj}}{\text{hyp}}\right) \qquad A = \tan^{-1}\left(\frac{\text{opp}}{\text{adj}}\right)$$



Example 1: Determine the measure of $\angle A$.



Example 2: Determine the measure of $\angle P$.



Ex. 3: In ΔPQR, $\angle Q = 90°$, $PR = 8$ cm and $PQ = 4$ cm. Find $\angle R$.

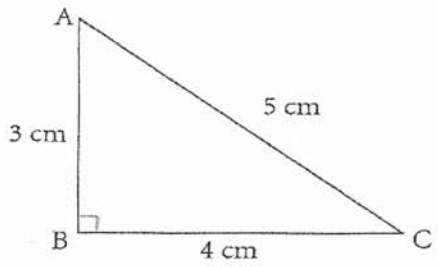
Determining the Measures of Angles in Right Triangles Worksheet

1. For $\triangle ABC$, write the ratio of sides for each trigonometric ratio.

   a) $\sin C$

   b) $\cos C$

   c) $\tan C$



2. Determine the measure of each angle to the nearest degree.

   a) $\sin X = 0.6735$ b) $\tan B = 1.674$ c) $\cos Z = 0.3176$

   d) $\sin K = 0.7419$ e) $\cos A = 0.5490$ f) $\tan G = 0.6203$

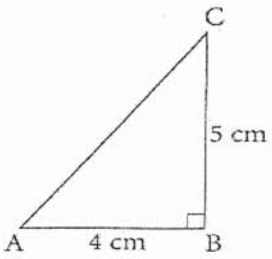
3. Determine the measure of each angle to the nearest degree.

   a) $\sin Y = \frac{5}{8}$ b) $\cos A = \frac{11}{15}$ c) $\sin C = \frac{5.2}{6.7}$

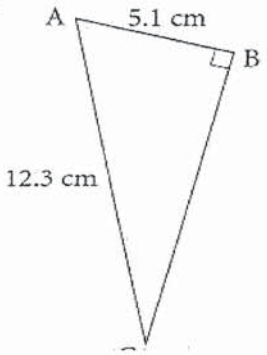
   d) $\tan K = \frac{4.3}{8.7}$ e) $\cos Z = \frac{3.6}{12.0}$ f) $\tan B = \frac{14.2}{9.6}$

4. Determine the measure of each $\angle A$ to the nearest degree.

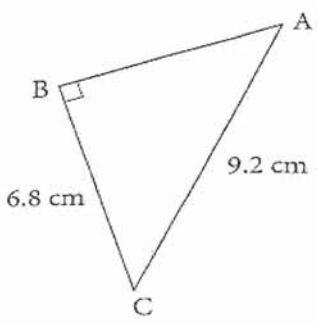
   a)



   b)

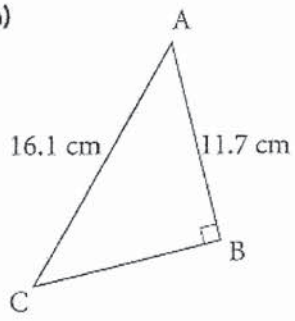


   c)

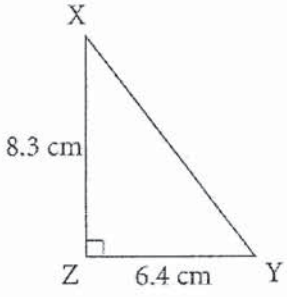


5. Determine the measures of the acute angles in each triangle to the nearest tenth of a degree.

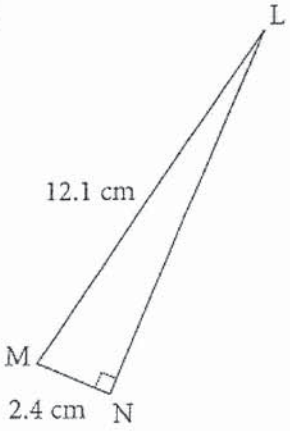
   a)



   b)

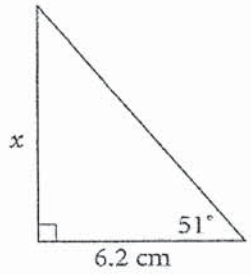


   c)

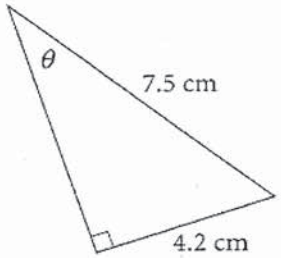


6. Determine the measure of the indicated side or angle. Give each measure to the nearest degree or tenth of a centimetre.

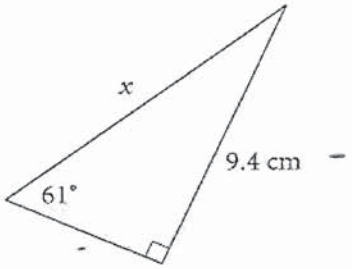
a)



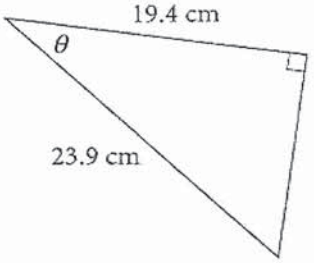
b)



c)

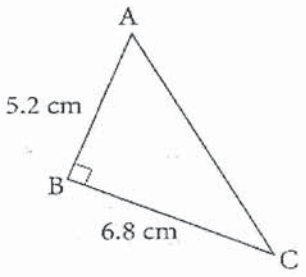


d)

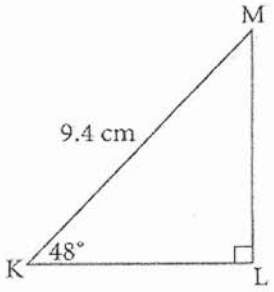


7. Solve each triangle. Give each measure to the nearest degree and/or the nearest unit.

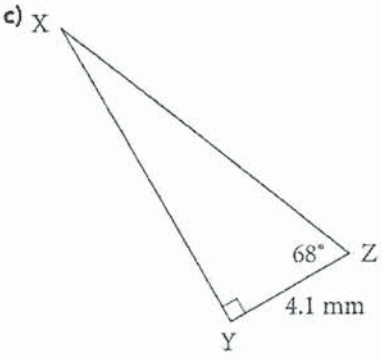
a)



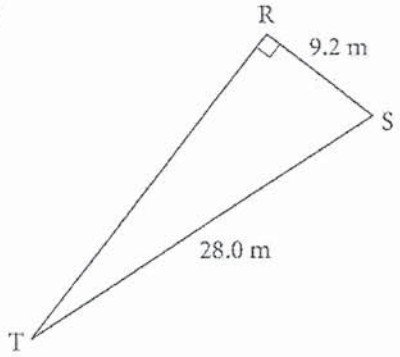
b)



c)



d)



8. Calculate each indicated angle to the nearest degree, and each indicated side to the nearest tenth of a unit.

a) Given $\triangle ABC$: $\angle B = 90°$, $a = 4.9$ m, and $c = 6.5$ m; calculate $\angle A$.

b) Given $\triangle XYZ$: $\angle Y = 90°$, $y = 9.2$ cm, and $z = 4.7$ cm; calculate $\angle X$.

c) Given $\triangle RST$: $\angle T = 90°$, $\angle S = 65°$, and $s = 7.4$ cm; calculate t.

d) Given $\triangle JKL$: $\angle K = 90°$, $k = 25.2$ mm, and $j = 11.3$ mm; calculate $\angle L$.

9. Knowledge/Understanding Solve each triangle. Round each angle to the nearest degree, and each length to the nearest tenth of a unit.

a) $\triangle ABC$: $\angle B = 90°$, $b = 34$ cm, and $a = 27$ cm

b) $\triangle JKL$: $\angle K = 90°$, $j = 37.8$ m, and $k = 41.5$ m

c) $\triangle PQR$: $\angle Q = 90°$, $r = 4.0$ cm, and $q = 5.0$ cm

d) $\triangle XYZ$: $\angle Y = 90°$, $z = 8.0$ cm, and $x = 4.0$ cm

10. Solve each triangle.

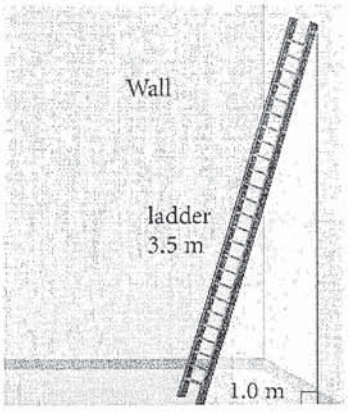
a) $\triangle RST$: $\angle T = 90°$, $\angle S = 54°$, and $r = 9.1$ mm

b) $\triangle LMN$: $\angle M = 90°$, $\angle L = 37°$, and $m = 15.6$ cm

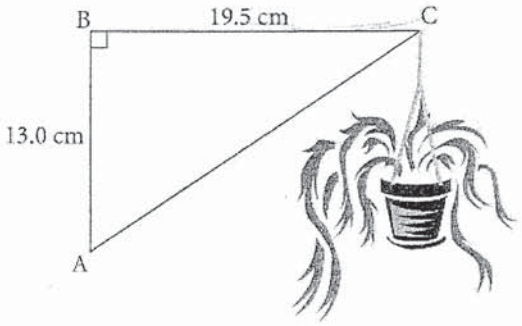
c) $\triangle ABC$: $\angle C = 90°$, $\angle B = 78°$, and $b = 4.9$ m

d) $\triangle DEF$: $\angle E = 90°$, $\angle F = 41°$, and $d = 4$ cm

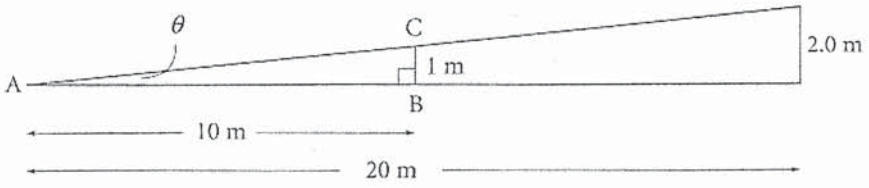
11. A ladder 3.5 m long is leaning against a wall. The foot of the ladder is 1.0 m from the wall. To the nearest degree, what angle does the ladder make with the ground?



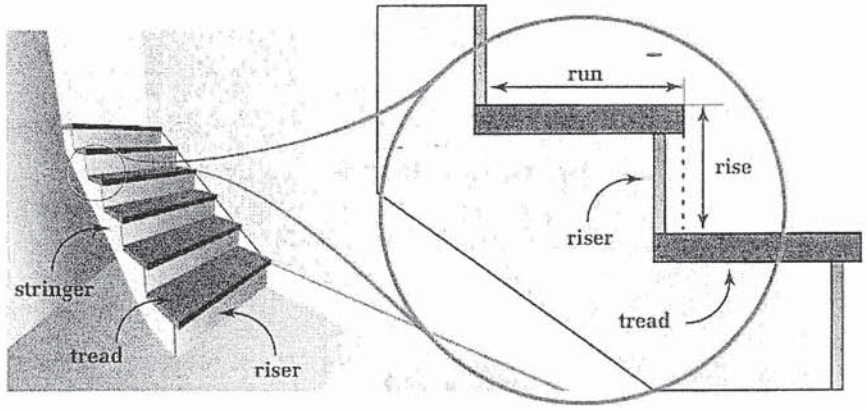
12. Application A bracket that holds a hanging plant is attached to a wall. The vertical and horizontal legs are 13.0 cm and 19.5 cm, respectively. What is the measure of angle A?



13. A ramp rises 1 m for every 10 m of horizontal distance. The ramp is inclined at an angle, θ, to the horizontal. Determine the measure of θ to the nearest degree.



14. A staircase has 3 main parts: the stringer, the riser, and the tread. A carpenter must obey local building codes when she constructs a staircase.



The code of one community requires that the sum of the riser and the tread must not exceed 449 mm. Calculate the angle of a staircase whose riser is 175 mm. Assume the measures are at the maximum allowed by the code.

15. **Communication** A helicopter is hovering 6000 m above the ground. A searchlight on the ground is a horizontal distance of 1700 m from the helicopter.

a) Draw a sketch of this situation.

b) The searchlight is aimed so its beam shines on the helicopter. Calculate the measure of the angle of elevation of the helicopter from the searchlight.

16. A cable car rises 150 m for every 50 m it travels horizontally. What angle does the cable make with level ground?

17. **Thinking/Inquiry/Problem Solving**

a) Use a ruler and grid paper. Draw a right $\triangle ABC$ for each ratio.

i) $\tan A = \frac{1}{2}$ **ii)** $\tan A = \frac{3}{4}$ **iii)** $\tan A = 1$

b) Calculate the measure of $\angle A$ for each triangle in part a.

c) Describe what happens to the measure of $\angle A$ and to the shape of $\triangle ABC$ as $\tan A$ increases.

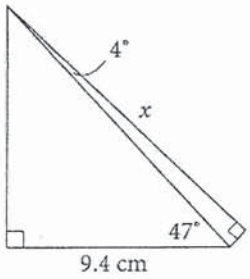
d) Predict the shape of $\triangle ABC$ and the measure of $\angle A$ when $\tan A = \frac{4}{3}$. Draw the triangle and calculate the measure of $\angle A$ to verify your prediction.

e) What is the greatest possible value of $\tan A$ in a right triangle? Explain.

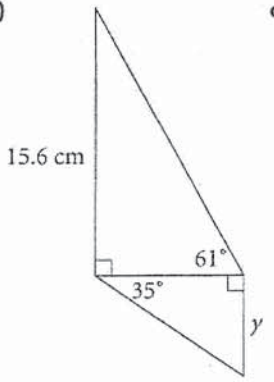
18. **Communication** When you solve a triangle, you calculate the measures of all the sides and angles. Why is it better to calculate using the given measures rather than previously calculated measures?

19. Determine the measure of each indicated side or angle.

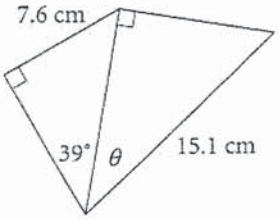
a)



b)



c)



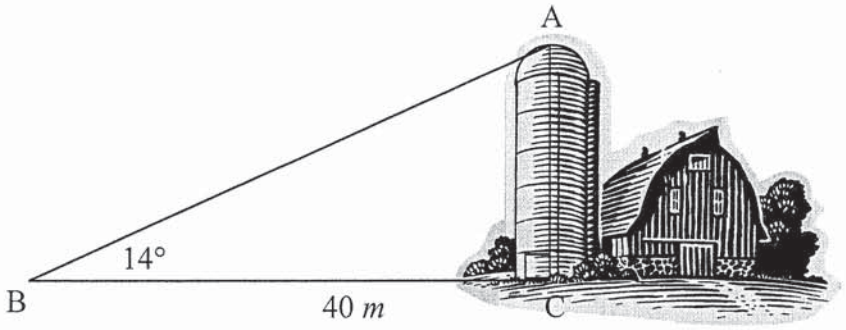
Answers

1. a) 3/5 b) 4/5 c) 3/4 2. a) 42° b) 59° c) 72° d) 48° e) 57° f) 32° 3. a) 39° b) 43° c) 51° d) 26° e) 73° f) 56° 4. a) 52° b) 66° c) 48° 5. a) A = 44°, C = 46° b) X = 38°, Y = 52° c) L = 11°, M = 79° 6. a) 7.66 cm b) 34° c) 10.8 cm d) 36° 7. a) b = 8.6 cm, A = 53°, C = 37° b) M = 42°, k = 7.0 cm, m = 6.3 cm c) X = 22°, z = 10.1 mm, y = 10.9 mm d) s = 26.4 m, S = 71°, T = 19° 8. a) 37° b) 59° c) 8.2 cm d) 63° 9. a) c = 20.7 cm, C = 37°, A = 53° b) l = 17.1 cm, L = 24°, 66° c) p = 3.0 cm, R = 53°, P = 37° d) y = 8.9 cm, Z = 63°, X = 27° 10. a) R = 36°, s = 12.5 cm, t = 15.5 cm b) N = 53°, n = 12.5 cm, l = 9.4 cm c) A = 12°. a = 1.04 m, c = 5.0 m d) D = 49°, 5.3 cm, f = 4.6 cm 11. 73° 12. 56° 13. 5.7° 14. 33° 15. b) 74° 16. 72° 19. a) 13.7 cm b) 19.7 cm c) 37°

Trigonometry Application Problems

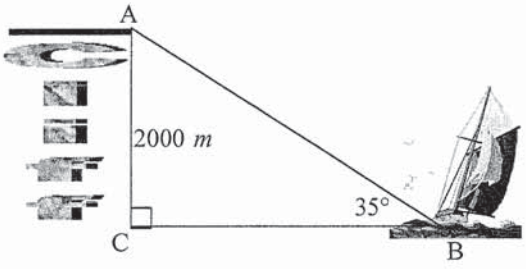
Problem 1:

While walking to school you pass a barn with a silo. Looking up to the top of the silo you estimate the angle of elevation to the top of the silo to be about 14°. You continue walking and find that you were around 40 m from the silo. Using this information and your knowledge of trigonometric ratios calculate the height of the silo.



Problem 2:

A sailboat is approaching a cliff. The angle of elevation from the sailboat to the top of the cliff is 35^0. The height of the cliff is known to be about 2000 m. How far is the sailboat away from the base of the cliff?



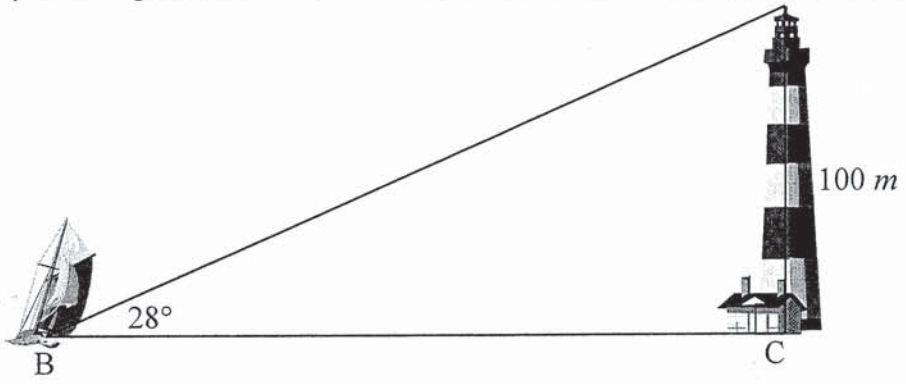
Problem 3:

A sailboat that is 2 km due west of a lighthouse sends a signal to the lighthouse that it is in distress. The lighthouse quickly signals a rescue plane that is 7 km due south of the lighthouse. What heading from due north should the plane take in order to intercept the troubled sailboat?

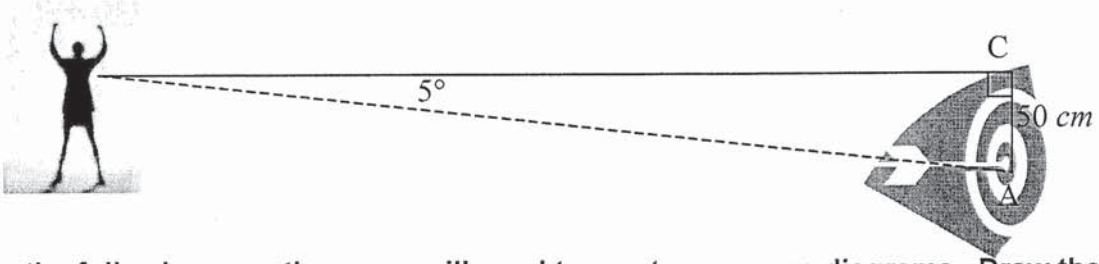
Trigonometry Applications Worksheet

Round ∠'s to whole degrees. Length answers should be rounded to 1 decimal place and include units.

1. The top of a lighthouse is 100 m above sea level. The angle of elevation from the deck of the sailboat to the top of the lighthouse is 28°. Calculate the distance between the sailboat and the lighthouse.



2. An archer shoots and gets a bulls-eye on the target. From the archer's eye level the angle of depression to the bulls-eye is 5°. The arrow is in the target 50 cm below the archer's eye level. Calculate the distance the arrow flew to hit the target (*the dotted line*).



For the following questions you will need to create your own diagrams. Draw them carefully and refer to the written description to ensure you find the correct solution.

3. Two islands **A** and **B** are 3 km apart. A third island **C** is located due south of **A** and due west of **B**. From island **B** the angle between islands **A** and **C** is 33°. Calculate how far island **C** is from island **A** and from island **B**.

4. The foot (*bottom*) of a ladder is placed 1.5 m from a wall. The ladder makes a 70° angle with the level ground. Find the length of the ladder. (Round to one decimal place.)

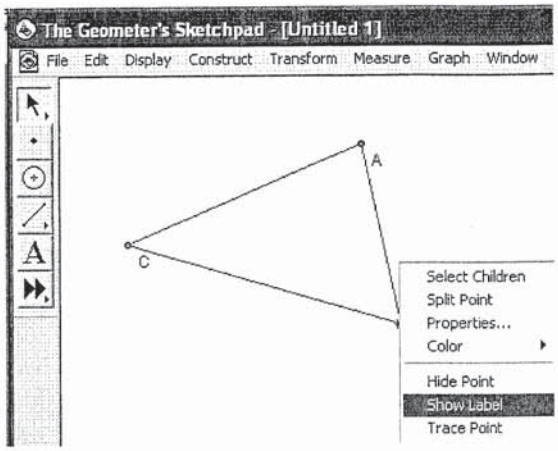
5. A tow truck raises the front end of a car 0.75 m above the ground. If the car is 2.8 m long what angle does the car make with the ground?

6. A construction engineer determines that a straight road must rise vertically 45 m over a 250 m distance measured along the surface of the road (this represents the hypotenuse of the right triangle). Calculate the angle of elevation of the road.

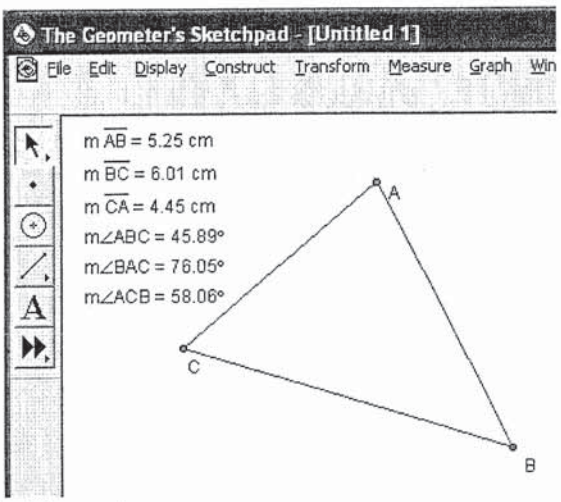
Solutions:

1. 188.1 *m* 2. 573.7 *cm* 3. Distance A to C: 1.6 *km* Distance B to C: 2.5 *km*
4. 4.4 *m* 5. 16° 6. 10°

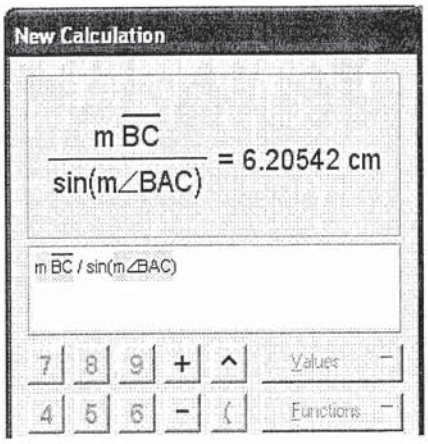
Investigation: Sine Law – Geometer's Sketchpad



1. Load Geometer's Sketchpad.
2. Start with a new document (*default*).
3. Select the Straightedge Tool (*4th button down the toolbar*)
4. Draw three lines – making sure that each new line starts from a previous line and that the last point returns to the first completing the triangle. (*shown right*)
5. Switch to the selection tool (*1st button on the toolbar*)
6. Select and right-click on each vertex and from the short-cut menu select "Show Label" (*also shown right*)



7. Next select any line and from the Measure menu (*or from the right-click short-cut*) select "Length". This should display **m $\overline{AB}$** (*shown*)
8. Repeat **Step 6** for the other lines, making sure to unselect before selecting a new line. (*If anything else is selected length may not appear on the menu.*)
9. Next select in the following order the vertices: A, B then C – then click the Measure menu and choose "Angle". This should display **m ∠ ABC** and the measure of that angle.
10. Now repeat **Step 8** but for angles ∠ BAC and ∠ ACB. (*shown*)
11. If you select any point you can drag the point to a new location and all of the measurements update automatically. (*You can also select and move an entire line.*)
12. Try this and adjust the position of the triangle to leave more room below our measurements.
13. We will now add some calculations namely the values for the Sine Law: $\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$
14. To do this select the Measure menu and select "Calculate...". A new dialogue box appears (*shown right*) where we will enter our calculation.
15. First click on the measurement for side *a* (*in this case it is* **m $\overline{BC}$**), then click on the division sign and type "sin" for the sine function, next click on the measurement for ∠ A (*in this case it is* **m ∠ BAC** (*depending on the size of your triangle*

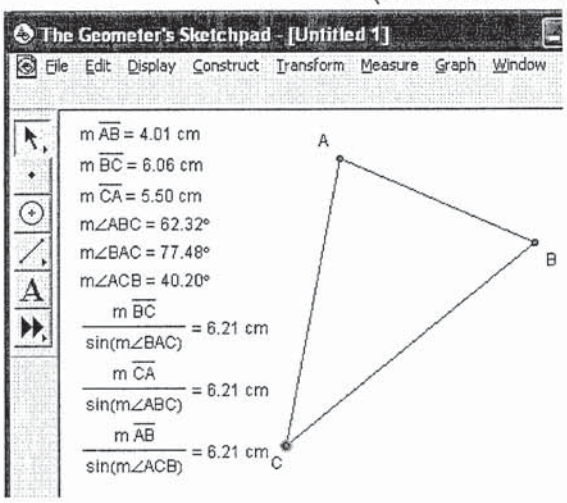


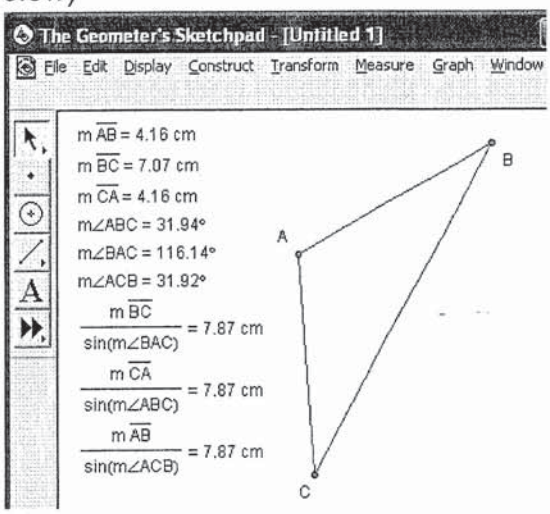
you will see different results.) Click **OK**

16. This will add a new measurement to your document, repeat step 15 for side b and side c. For side b use **m $\overline{CA}$** and sin(**m** $\angle$ **ABC)** for side c use **m $\overline{AB}$** and sin(**m** $\angle$ **ACB**). Calculations are shown in the bottom diagrams.

17. Now change the position of your vertices; this will change the lengths and angles in your triangle – make note of what happens to all three of the calculation boxes for the Sine Law:

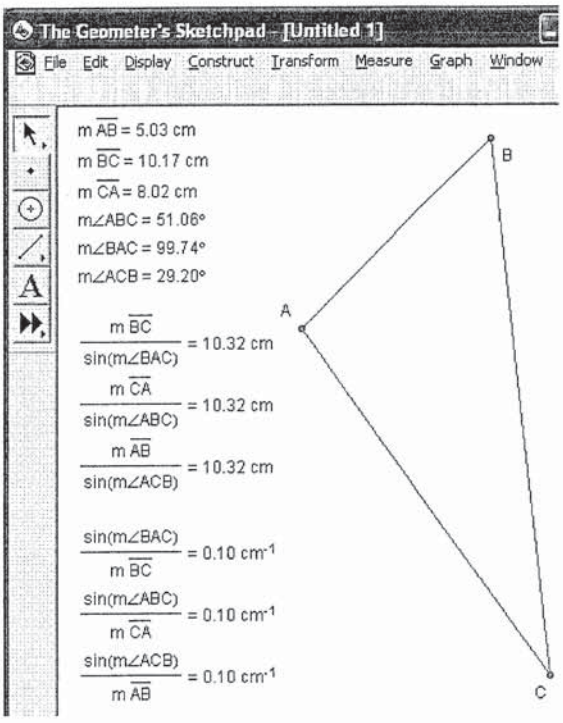
$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$. *(two variations shown below)*





18. Next create three more calculations for the other version of the Sine Law:

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$ *(shown right)*



19. Experiment with more positions of the triangle vertices.

20. Notice that the set of three values in either version of the Sine Law remain the same. This shows that the ratio of any side to the sine of the corresponding angle in a triangle remains equal to the ratio of any other side to the sine of the corresponding angle. Either

$$\frac{\sin \angle A}{a} = \frac{\sin \angle B}{b} = \frac{\sin \angle C}{c}$$

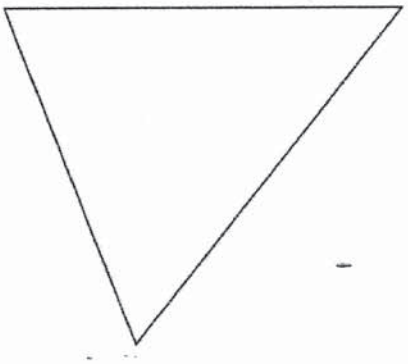
or

$$\frac{a}{\sin \angle A} = \frac{b}{\sin \angle B} = \frac{c}{\sin \angle C}$$

The Sine Law - Finding the Sides

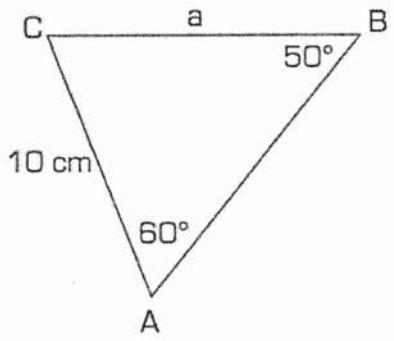
The Sine Law:

- The relationship between the length of the sides and the opposite angles in any triangle.
- It is used when you are given 2 angles and a side

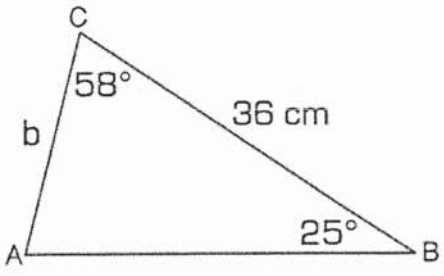


Example 1. Find the indicated side.

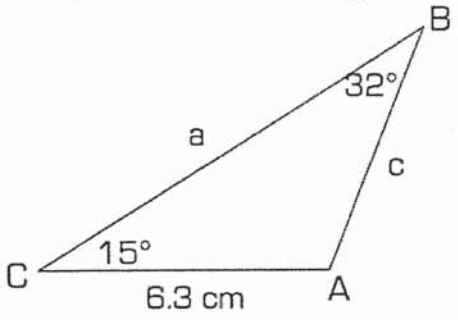
a)



b)



Example 2. Solve the triangle.



Example 3.
A cottage under construction is to be 12.6 m wide. The two sides of the roof are to be supported by rafters of equal length that meet at a 50° angle. How long should the rafters be?

The Sine Law - Calculating Angles

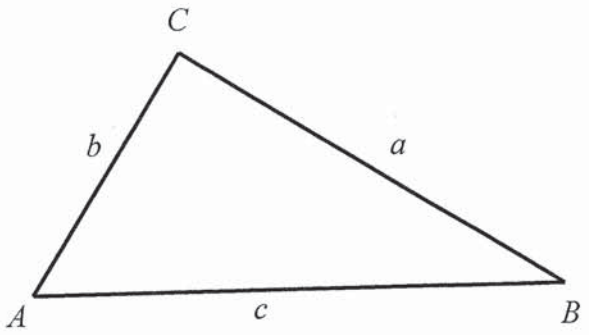
It is used to solve non-right triangles: the triangle does not contain a 90° angle.

It is used when you are given two angles and a side **(ASA)**, or two sides and an opposite angle **(SSA)**.

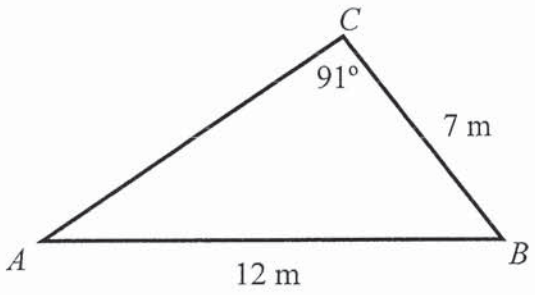
The Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{(for sides)}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \quad \text{(for angles)}$$



Example 1: Determine the measure of ∠A.



Example 2: Given ΔPQR with ∠Q = 44°, q = 73 cm and p = 95 cm, find the measure of ∠P.

Example 3: Given ΔSUM with $\angle S = 41°$, $m = 52.5$ cm $s = 35.2$ cm. Solve the triangle. (Find all the unknown sides and angles)

The Sine Law in Acute Triangles Worksheet

A

1. Given each expression for side a, calculate its measure. Give your answers to 1 decimal place.

   a) $a = \frac{27 \sin 62^\circ}{\sin 34^\circ}$ b) $a = \frac{3 \sin 70^\circ}{\sin 60^\circ}$ c) $a = \frac{13.4 \sin 55^\circ}{\sin 48^\circ}$

2. Given each value of sin P, calculate the measure of acute $\angle$P to the nearest degree.

   a) $\sin P = 0.79$ b) $\sin P = 0.65$ c) $\sin P = \frac{3}{4}$

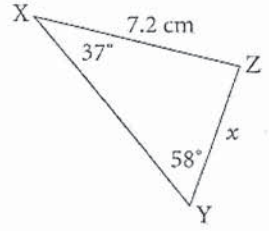
3. Given each expression for sin B, calculate the measure of acute $\angle$B.

   a) $\sin B = \frac{3 \sin 50^\circ}{5}$ b) $\sin B = \frac{87 \sin 43^\circ}{74}$ c) $\sin B = \frac{1.2 \sin 12^\circ}{3.1}$

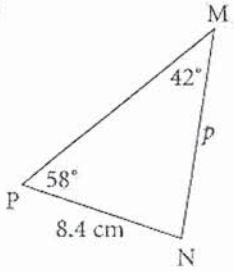
B

4. Determine the length of each indicated side. Round your answers to the nearest 0.1 cm.

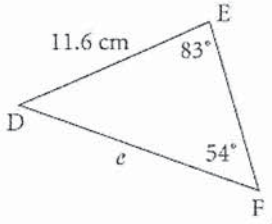
   a)



   b)

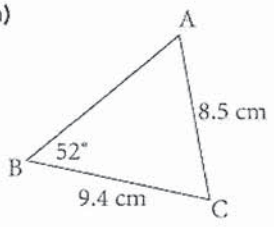


   c)

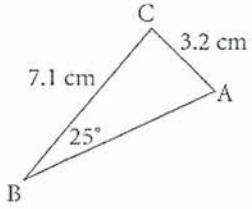


5. In each acute $\triangle$ABC, determine the measure of acute $\angle$A to the nearest degree.

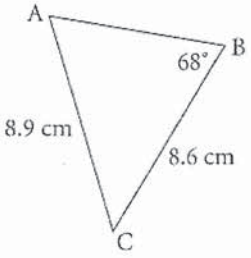
   a)



   b)

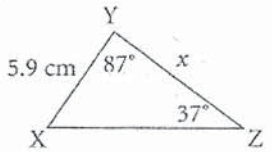


   c)

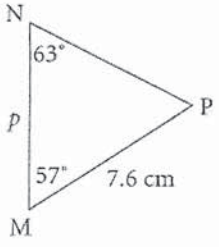


6. Determine the length of each indicated side. Answer to the nearest tenth of a unit.

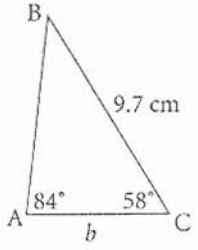
   a)



   b)

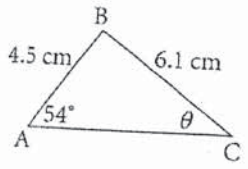


   c)

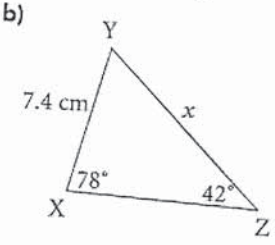


7. Determine the measure of each indicated side or acute angle. Give the lengths to one decimal place, and the angles to the nearest degree.

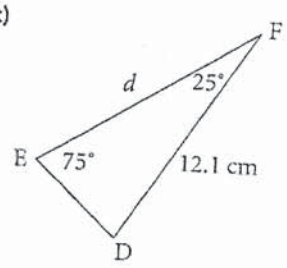
a)



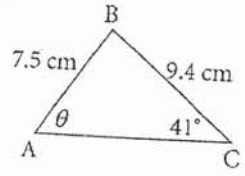
b)



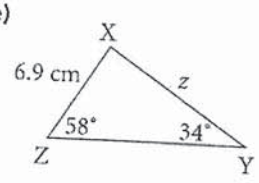
c)



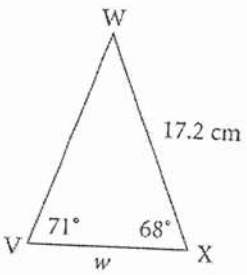
d)



e)

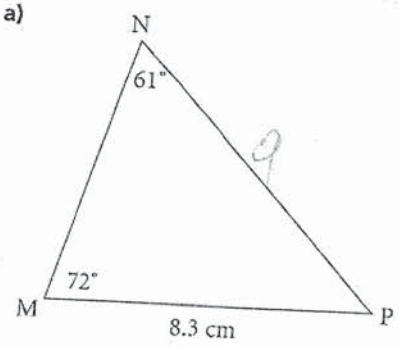


f)

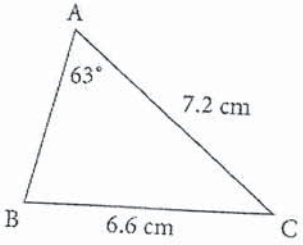


8. Knowledge/Understanding Solve each acute triangle.

a)



b)



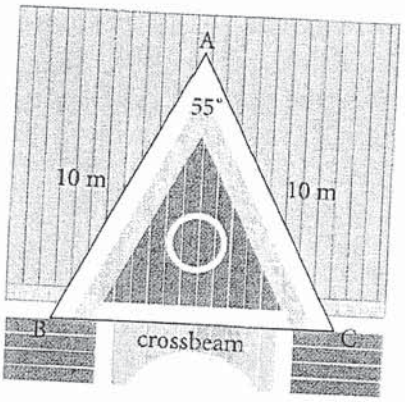
9. Solve each acute triangle.

a) $\triangle ABC$ given $\angle B = 63°$, $\angle C = 47°$, and $b = 9.2$ cm

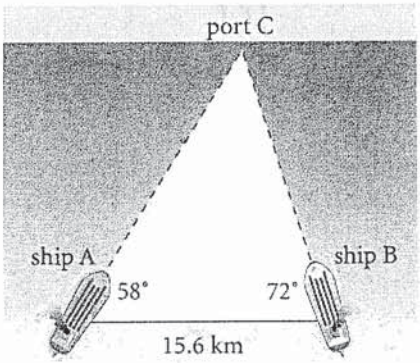
b) $\triangle PQR$ given $\angle P = 42°$, $\angle Q = 53°$, and $r = 7.7$ cm

c) $\triangle KMN$ given $\angle K = 55°$, $\angle N = 55°$, and $k = 6.2$ cm

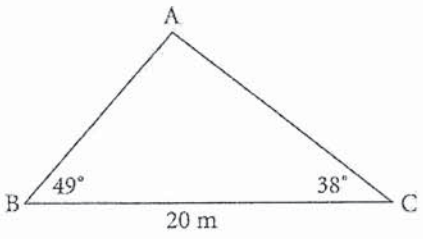
10. Application An A-frame building has a roof as shown. The angle at the vertex of the A-frame is 55°. The distance from the vertex of the roof to the endpoints of the crossbeam is 10 m. Calculate the length of the crossbeam.

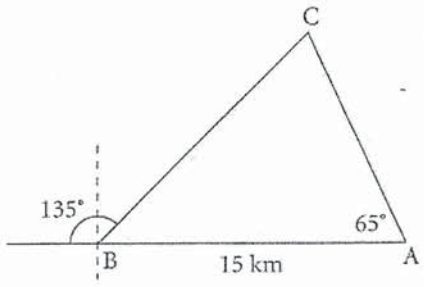


11. Ships A and B at sea are 15.6 km apart. A port, C, can be seen from the deck of each ship. The angles between the line joining the ships and the lines of sight to the port are 58° and 72°, respectively. How far is each ship from the port?



12. **Thinking/Inquiry/Problem Solving**
A landscape designer is making a triangular garden (below left). The garden is to be enclosed by a fence costing $1.50/m. What will be the cost of the fence?





13. A boat leaves a dock at point A on a lake and travels 15 km due west to point B on the lake. The boat then makes a 135° turn and travels to a dock at point C (above right). Line AC makes an angle of 65° with AB. What is the shortest distance between the dock at C and the dock at A?

14. Three straight roads join three towns, A, B, and C. The road between towns A and B is 35 km long. The road between towns A and C makes an angle of 70° with the road between towns A and B, and an angle of 60° with the road between towns C and B. A person drives from A to B to C. Determine the total distance driven.

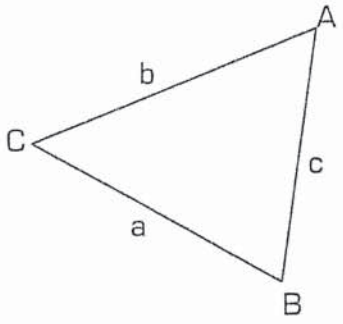
15. **Communication** Look at the exercises you have completed. Assume you know three measures in a triangle. Explain the conditions under which the Sine Law may be used to calculate the length of a side and the measure of an angle.

Answers

1. a) 42.6 b) 3.26 c) 14.8 2. a) 52° b) 41° c) 49° 3. a) 28° b) 53° c) 5° 4. a) 5.1 cm b) 10.6 cm c) 14.2 cm 5. a) 61° b) 70° c) 64° 6. a) 8.1 cm b) 7.4 cm c) 6.0 cm 7. a) 37° b) 10.8 cm c) 12.3 cm d) 55° e) 10.5 cm f) 11.9 cm 8. a) $\angle P = 47°$, m = 9.0 cm, p = 6.6 cm b) $\angle B = 76°$, $\angle C = 41°$, c = 4.8 cm 9. a) $\angle A = 70°$, c = 7.6 cm, a = 9.7 cm b) $\angle R = 85°$, q = 6.2 cm, p = 5.2 cm c) $\angle M = 70°$, m = 7.6 cm, n = 6.6 cm 10. 9.2 m 11. Ship A = 19.4 km, Ship B = 17.3 km 12. $71.10 13. 11.3 km 14. 117.5 km

The Cosine Law

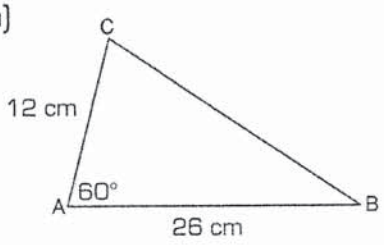
The second method to solve non-right triangles is the cosine law. You use the cosine law to calculate sides when you are given 2 sides and the contained angle (ie. the angle the two sides make).



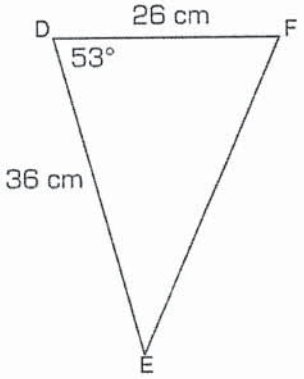
$$c^2 = a^2 + b^2 - 2ab\cos C$$

Ex 1. Find the missing side.

a)



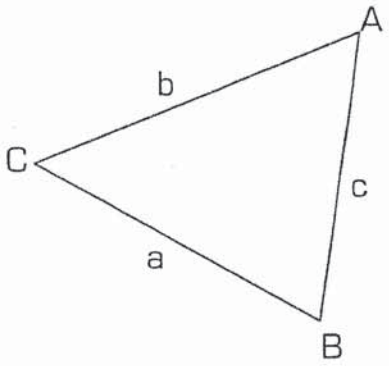
b)



Ex 2. Solve triangle ΔMNO given, ∠O = 110°, m = 22.4 cm and n = 14.2 cm.

The Cosine Law - Solving Angles

When calculating angles of a non-right triangle, if all 3 sides(SSS) are given use cosine law.

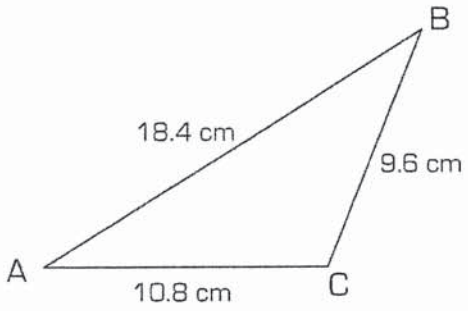


$$c^2 = a^2 + b^2 - 2ab\cos C$$

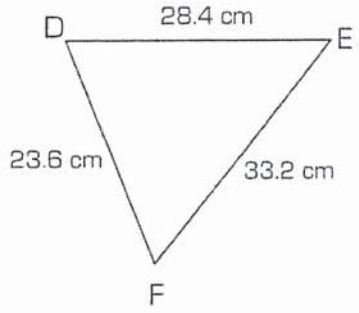
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Ex 1. Find all angles for each triangle.

a)



b)



2. A hockey net is 2 m wide. A player shoots from a point where the puck is 3.2 m from one goal post and 4.4 m from the other. Within what angle must he make his shot to hit the net?

The Cosine Law Worksheet

A

1. Given each expression for side b, calculate its measure.
   a) $b^2 = 9^2 + 3^2 - 2(9)(3)\cos 23°$
   b) $b^2 = 62^2 + 56^2 - 2(62)(56)\cos 37°$
   c) $b^2 = 16.1^2 + 20.3^2 - 2(16.1)(20.3)\cos 42°$
   d) $b^2 = 5.6^2 + 7.2^2 - 2(5.6)(7.2)\cos 113°$

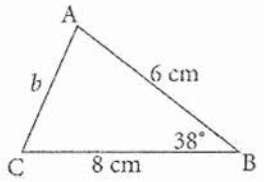
2. Given each value of cos A, calculate the measure of $\angle A$.
   a) $\cos A = \frac{4}{5}$
   b) $\cos A = 0.5796$
   c) $\cos A = -0.3824$
   d) $\cos A = -0.4313$
   e) $\cos A = 0.1144$
   f) $\cos A = -0.9899$

3. Given each expression for cos C, calculate the measure of $\angle C$.
   a) $\cos C = \frac{62^2 + 53^2 - 38^2}{2(62)(53)}$
   b) $\cos C = \frac{45^2 + 32^2 - 50^2}{2(45)(32)}$
   c) $\cos C = \frac{5^2 + 3^2 - 7^2}{2(5)(3)}$
   d) $\cos C = \frac{4.1^2 + 2.6^2 - 5.5^2}{2(4.1)(2.6)}$

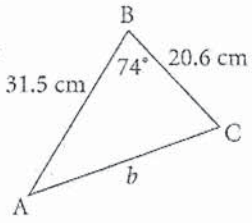
B

4. Calculate the length of each side b.

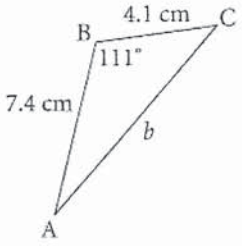
a)



b)

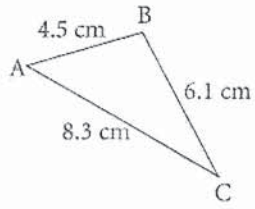


c)

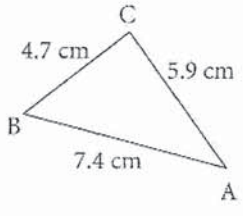


5. Determine the measure of $\angle A$ in each triangle.

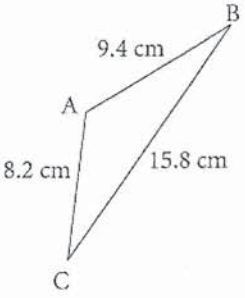
a)



b)

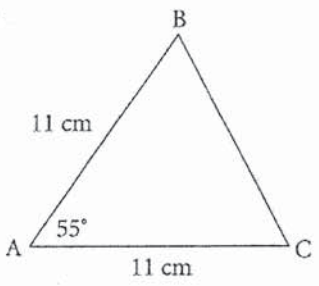


c)

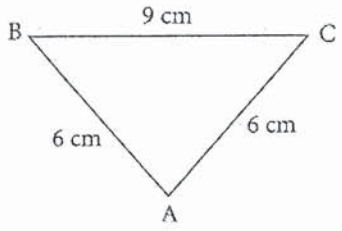


6. Knowledge/Understanding Solve each $\triangle ABC$.

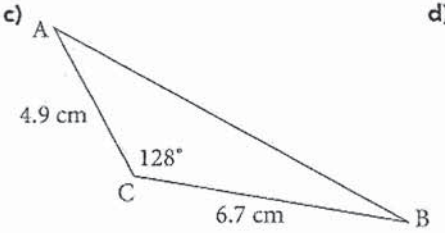
a)



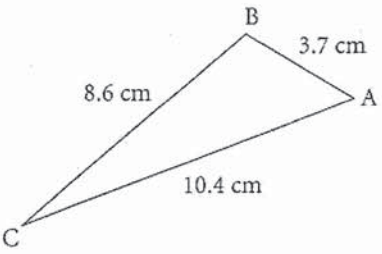
b)



c)



d)



7. Solve each triangle.

a) $\triangle ABC$ given $\angle B = 68°$, $c = 15.7$ cm, and $a = 18.4$ cm

b) $\triangle DEF$ given $\angle D = 130°$, $e = 9.8$ cm, and $f = 9.4$ cm

c) $\triangle RST$ given $\angle S = 118°$, $r = 8.5$ cm, and $t = 5.4$ cm

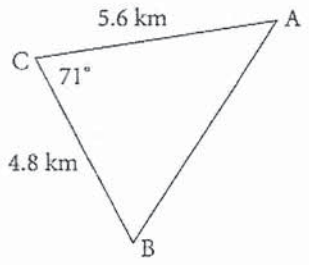
8. Solve each triangle.

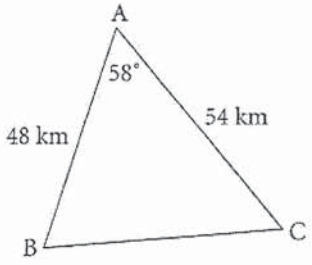
a) $\triangle KMN$ given $k = 5.2$ cm, $m = 2.7$ cm, and $n = 6.5$ cm

b) $\triangle PQR$ given $q = 3.1$ cm, $r = 1.9$ cm, and $p = 4.7$ cm

c) $\triangle XYZ$ given $x = 9.6$ cm, $y = 8.2$ cm, and $z = 4.8$ cm

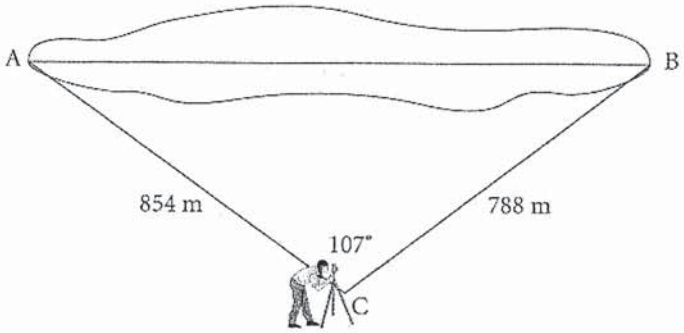
9. From a point, C, on shore, an observer sees two ships at sea. Ship A is 5.6 km from the observer and ship B is 4.8 km from the observer. The angle between the sight lines to the ships is 71° (below left). Determine the distance between the ships.





10. Two straight roads intersect in town A at an angle of 58°. The distance from town A to town B is 48 km. The distance from town A to town C is 54 km (above right). A straight road, connecting town B to town C, is under construction. Determine the length of the new road.

11. Application To determine the length of a small lake, a surveyor takes measurements. From a point, C, the distances to points A and B at the ends of the lake are 854 m and 788 m. The angle between the lines of sight to the two points is 107°. Determine the length of the lake.



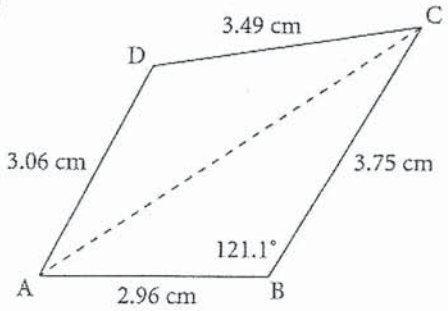
12. A hockey net is 1.8 m wide. A player is 6.0 m from one goalpost and 6.7 m from the other. Within what angle must she keep her shot in order to score a goal?

13. A lighthouse is located at point A. A ship travels from point B to point C. At point B, the distance between the ship and the lighthouse is 7.5 km. At point C, the distance between the ship and the lighthouse is 8.6 km. Angle BAC is 58°. Determine the distance between B and C.

14. Communication Do the Sine Law and the Cosine Law apply to right triangles? Include examples in your explanation.

15. In the Canadian Football League, the distance from one upright to the other upright on the goalposts is 6.16 yards. A field goal kicker kicks from a point 41 yd. from one upright and 41.5 yd. from the other. Within what angle must he kick the football in order to make the field goal?

16. Thinking/Inquiry/Problem Solving In quadrilateral ABCD, AB = 2.96 cm, BC = 3.75 cm, CD = 3.49 cm, AD = 3.06 cm, and ∠B = 121.1°. Calculate the measure of ∠D.



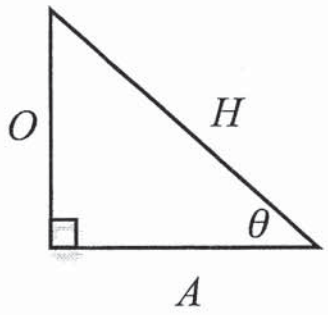
Answers

1. a) 6.35 b) 37.9 c) 13.6 d) 10.7 2. a) 37° b) 55° c) 112° d) 116° e) 83° f) 172° 3. a) 36° b) 79° c) 120° d) 108°
4. a) 4.9 cm b) 32.5 cm c) 9.7 cm 5. a) 46° b) 39° c) 128° 6. a) a = 10.2 cm, ∠B = 62.5°, ∠C = 62.5° b) ∠A = 97°, ∠B = 41°, ∠C = 42°
c) c = 10.5 cm, ∠B = 22°, ∠A = 30° d) ∠A = 52°, ∠B = 109°, ∠C = 19° 7. a) b = 19.2 cm, ∠A = 63°, ∠C = 49° b) d = 17.4 cm, ∠F = 24°, ∠E = 26°
c) s = 12.0 cm, ∠R = 37°, ∠T = 23° 8. a) ∠N = 106°, ∠K = 50°, ∠M = 24° b) ∠P = 139°, ∠Q = 26°, ∠R = 15° c) ∠X = 91°, ∠Y = 59°, ∠L = 30°
9. 6.07 km 10. 49.73 km 11. 1320.52 km 12. 15° 13. 7.86 km 15. 9° 16. 127°

Trigonometry Ratios and Laws: Choosing the Right Method

Trig Ratios (SOH CAH TOA)

- Used for right triangles



$$\sin\theta = \frac{O}{H} \qquad \cos\theta = \frac{A}{H} \qquad \tan\theta = \frac{O}{A}$$

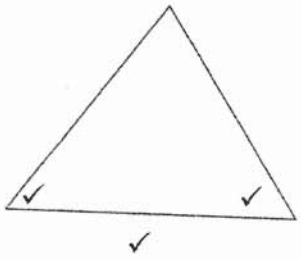
$$\theta = \sin^{-1}\left(\frac{O}{H}\right) \qquad \theta = \cos^{-1}\left(\frac{A}{H}\right) \qquad \theta = \tan^{-1}\left(\frac{O}{A}\right)$$

The Sine Law

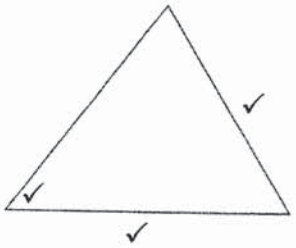
- Used for non-right triangles

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

⇒ Used to calculate sides
⇒ Given: 2 angles and a side (ASA)



⇒ Used to calculate angles
⇒ Given: 2 sides and an opposite angle (SSA)

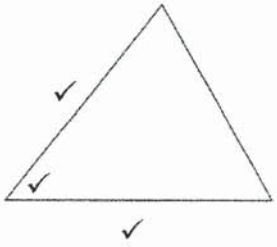


The Cosine Law

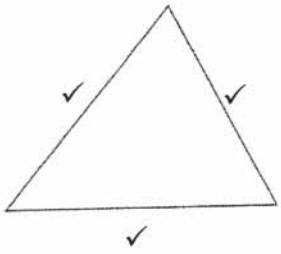
- Used for non-right triangles

$$c^2 = a^2 + b^2 - 2ab\cos C \qquad \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

⇒ Used to calculate sides
⇒ Given: 2 sides and the contained angle (SAS)



⇒ Used to calculate angles
⇒ Given: all 3 sides (SSS)

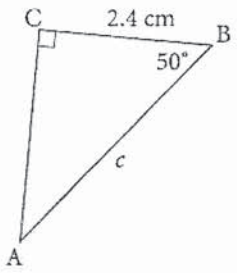


Selecting a Strategy Worksheet

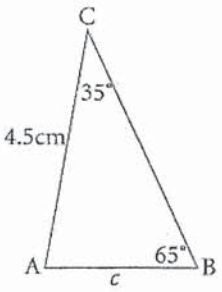
A

1. Calculate length c in each triangle.

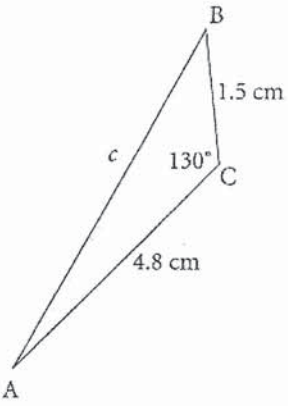
a)



b)



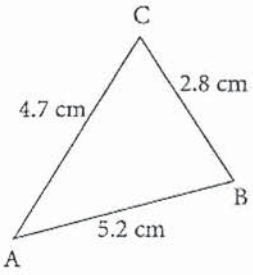
c)



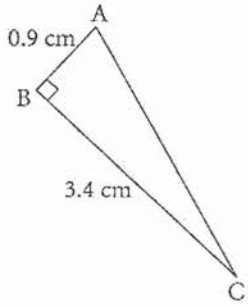
B

2. Calculate the measure of $\angle C$ in each triangle.

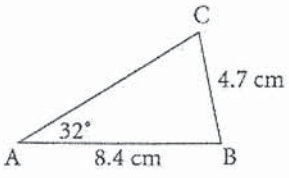
a)



b)



c)



3. **Knowledge/Understanding** Solve each triangle.

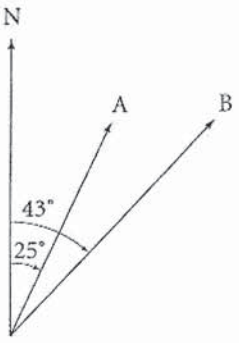
a) $\triangle ABC$ given $\angle A = 42°$, $\angle C = 34°$, and $c = 12.0$ cm

b) $\triangle PQR$ given $\angle R = 112°$, $p = 31.2$ cm, and $r = 45.0$ cm

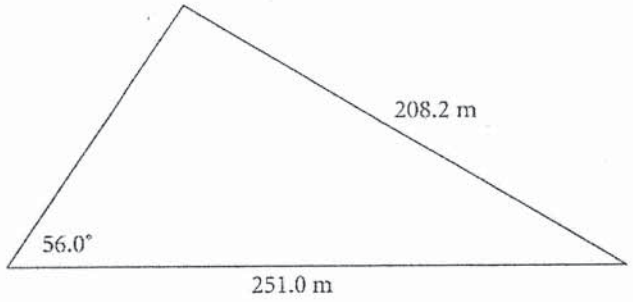
c) $\triangle ABC$ given $\angle A = 78°$, $\angle B = 90°$, and $a = 5.9$ cm

4. **Communication** Choose one part of exercise 3. Describe your strategy for solving the triangle.

5. Two ships leave port at the same time. One sails at 17 km/h on a bearing of 25°. The other ship sails at 21 km/h on a bearing of 43°. How far apart are the ships after 2 h?

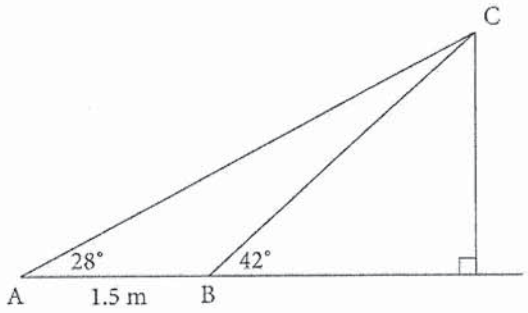


6. The longer sides of a triangular park measure 251.0 m and 208.2 m, respectively. The angle between the longest and shortest side is 56.0°.
   a) What is the angle between the shorter two sides of the park?
   b) How long is the shortest side?



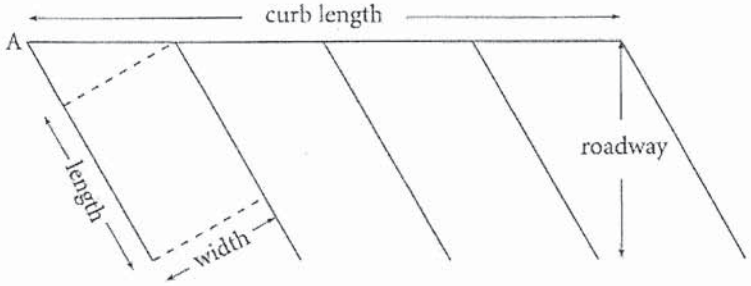
7. A surveyor's assistant stands on horizontal ground 34 m from the base of a building. She measures the angle of elevation to the top of the building to be 37°. Her eye level is 1.5 m. Calculate the height of the building.

8. **Application** A tower is supported by two guy wires, both attached to the ground on the same side of the tower. The angle of inclination of the first guy wire is 42° and the angle of inclination of the second guy wire, 1.5 m away, is 28° (below left).
   a) What is the measure of ∠ABC?
   b) Determine the length of each guy wire.



9. Three towns, A, B, and C, are joined by three straight roads. The road between towns A and C is 140 km long and makes an angle of 30° with the road between towns A and B, and an angle of 18° with the road between towns C and B.
   a) Without doing any calculations, predict which pair of towns is closest together.
   b) Verify your prediction by calculation.

10. Thinking/Inquiry/Problem Solving Angle parking allows more cars to park along a street than does parallel parking. However, the cars use more of the street width when angle parked.

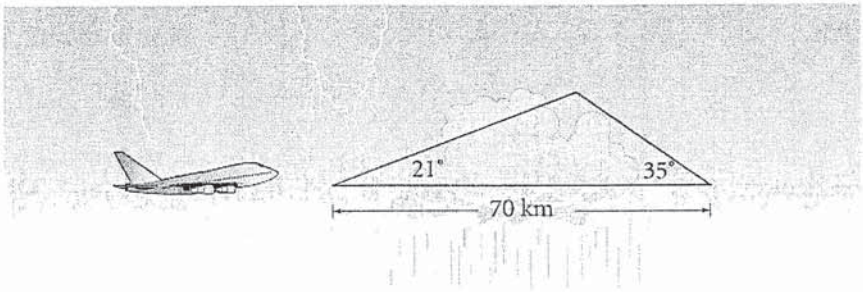


a) Each car requires a space 2.7 m wide. Determine the curb length required to park 20 cars when $\angle A = 50°$.

b) Suppose 20 cars had to be parked in 60 m of curb length. Determine the measure of $\angle A$.

c) Each car requires a space 6.5 m long. Determine the width of roadway used for parking when $\angle A = 50°$.

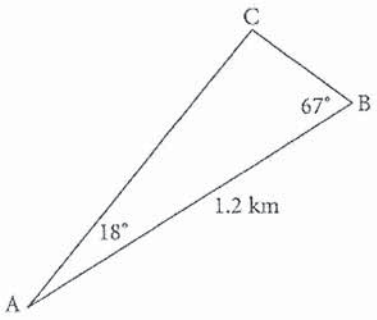
11. The pilot of an airplane encounters a group of thundershowers. To detour around these, she turns at an angle of 21° to the original path, then flies for a while. She turns the airplane again to intercept her original path at an angle of 35°, 70 km from where she left it. How much further did she travel because of the detour?



12. Two cabins, A and B, are located 1.2 km apart on the same side of a river. A boat launch is located across the river at C. Angles A and B are 18° and 67° respectively.

a) Determine the distance of each cabin from the boat launch.

b) Calculate the width of the river.



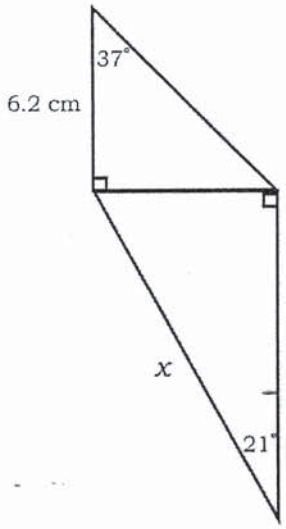
Answers

1. a) 3.7 cm b) 2.8 cm c) 5.8 cm 2. a) 96° b) 14.8° c) 71.3° 3a) 104°, 14.3 cm, 20.8 cm b) 40°, 28°, 22.8 cm c) 12°, 1.3 cm, 28.4 cm
4. Answers will vary 5. 18°, 34 km, 14.3 km 6a) 88° b) 147.6 m 7. 27 m 8a) 138° b) 2.9 m, 4.1 m 9a) towns A and B
b) AB = 58 km, CB = 94 km 10a) 70.5 m b) 64° c) 6.7 m 11. 9 km 12a) cabin A: 1.1 km, cabin B: 0.37 km b) 0.34 km

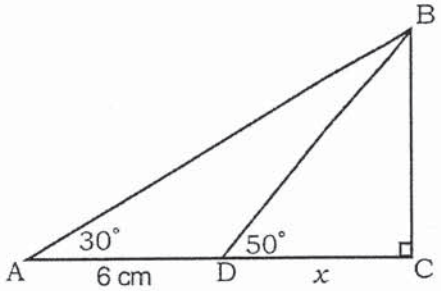
two-step trig questions

Ex 1. Find x.

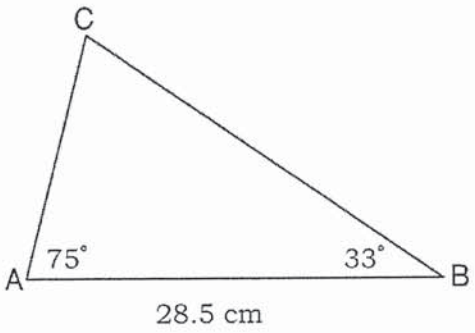
a)



b)



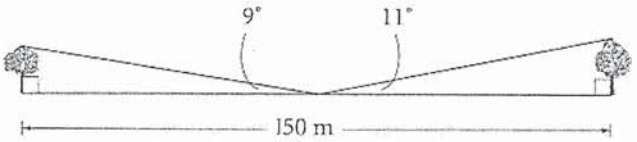
Ex 2. Calculate the area of the triangle given.



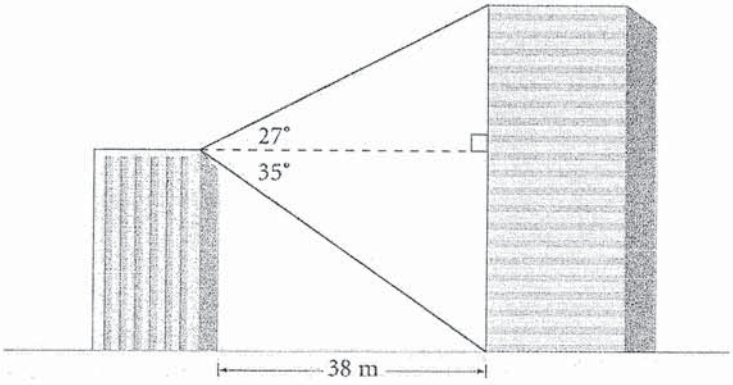
Two-Step Word Problems Worksheet

B

1. Two trees are 150 m apart. From a point halfway between the trees, Arturo measures the angles of elevation of their tops as 9° and 11°. How tall are the trees?

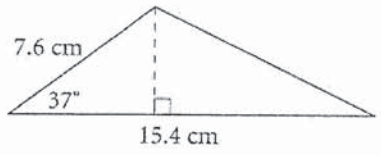


2. **Knowledge/Understanding** Two buildings are 38 m apart. From the top of the shorter building, the angle of elevation of the top of the taller building is 27° and the angle of depression of the base is 35°. Determine the height of each building.

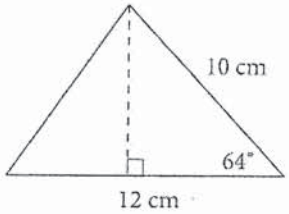


3. The area of a triangle is given by $A = \frac{1}{2} \times \text{Base} \times \text{Height}$. Determine the area of each triangle.

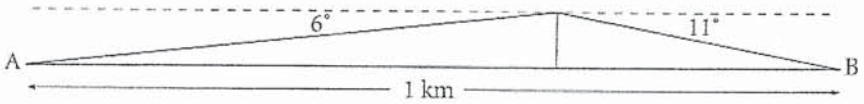
a)



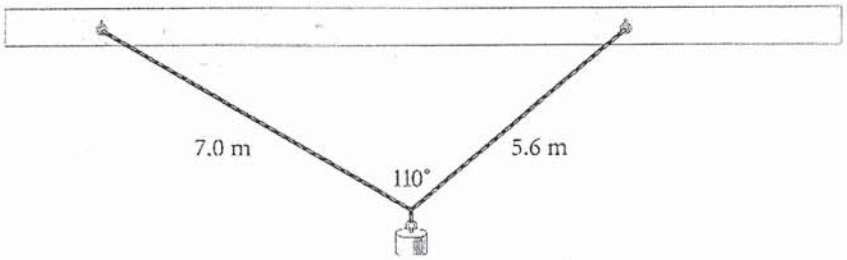
b)



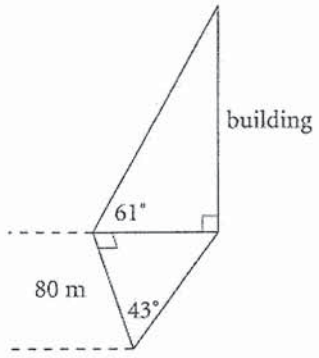
4. **Application** From an observation tower that overlooks a small lake, the angles of depression of point A, on one side of the lake, and point B, on the opposite side of the lake, are 6° and 11°, respectively. The points and the tower are in the same vertical plane and the distance from A to B is 1 km. Determine the height of the tower.



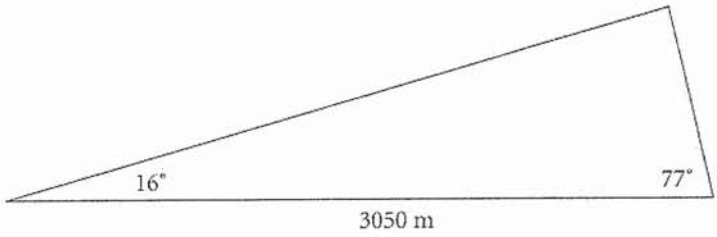
5. Two chains support a weight at the same point. The chains are attached to the ceiling and are 7.0 m and 5.6 m in length. The angle between the chains is 110°. How far apart are the chains at the ceiling?



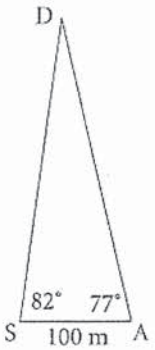
6. Calculate the height of the building in the diagram below.



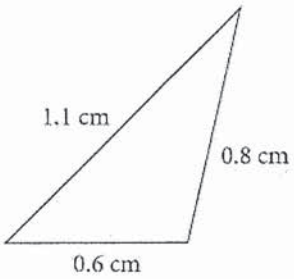
7. To calculate the height of the clouds one night, Amir and Suraya stood 3050 m apart. Amir shone a light on the clouds and Suraya measured the angle of elevation of the light reflecting from the clouds as 16°. The angle of inclination of the beam of light was 77°. What was the height of the clouds?



8. David and Susan want to calculate the distance between their cottages. The cottages are on opposite sides of a lake. They mark a point, A, 100 m along the shore from Susan's cottage. They measure ∠DSA as 82° and ∠SAD as 77°. Determine the distance between their cottages.



9. The edges of a saw tooth are 1.1 cm and 0.8 cm long. The base of the tooth is 0.6 cm long. Determine the angle at which the edges meet.



10. Communication From a certain point, the angle of elevation of the top of a tower is 31°. Another point, in the same vertical plane as the original point and the tower, is 130 m closer to the tower. From this point, the angle of elevation of the top of the tower is 38°.

a) Draw a diagram.

b) Describe how to calculate the height of the tower.

11. From the top of a lighthouse on a cliff, the angles of depression of two boats in the same vertical plane are measured as 5.4° and 3.6°. The boats are 1000 m apart. Determine the height of the top of the lighthouse above water level.

12. A children's playground is triangular. The sides of the playground measure 200 m, 250 m, and 300 m. Calculate the area of the playground.

13. A baseball diamond is a square with sides 27.4 m. The pitcher's mound is 18.4 m from home plate on the line joining home plate and second base. How far is the pitcher's mound from first base?

Answers

1. 12 m, 15 m 2. 27 m, 46 m 3a) 35 cm² b) 54 cm² 4. 68 m 5. 10 m 6. 140 m 7. 820 m 8. 270 m 9. 32°
11. 188 m 12. 25 000 m² 13. 19.4 m

Review Exercises

MATHEMATICS TOOLKIT

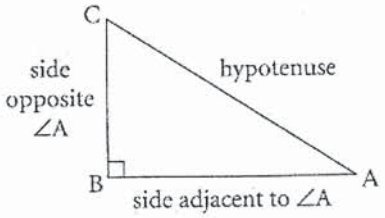
Trigonometry Tools

Trigonometric Ratios of Acute Angles

$$\sin A = \frac{\text{Length of side opposite } \angle A}{\text{Length of hypotenuse}}$$

$$\cos A = \frac{\text{Length of side adjacent to } \angle A}{\text{Length of hypotenuse}}$$

$$\tan A = \frac{\text{Length of side opposite } \angle A}{\text{Length of side adjacent to } \angle A}$$



Trigonometric Ratios of Angles between 0° and 180°

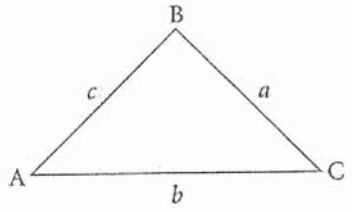
> For any acute angle A:

$\sin A = \sin(180° - A)$
$\cos A = -\cos(180° - A)$
$\tan A = -\tan(180° - A)$

The Sine Law

> In any △ABC:

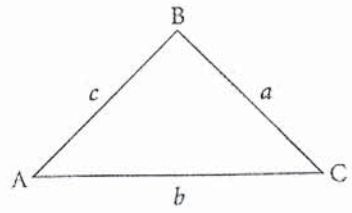
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$



The Cosine Law

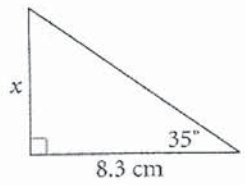
> In any △ABC:

$a^2 = b^2 + c^2 - 2bc\cos A$
$b^2 = a^2 + c^2 - 2ac\cos B$
$c^2 = a^2 + b^2 - 2ab\cos C$

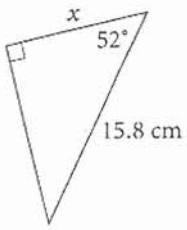


1. Determine length of each side x.

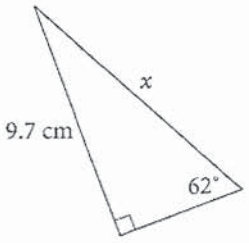
a)



b)



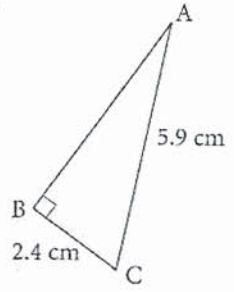
c)



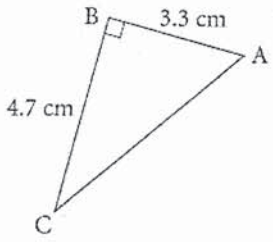
2. A guy wire is anchored to the ground 18 m from the base of a tower. The guy wire makes an angle of 71° with the ground. How high up the tower is the guy wire attached?

3. Determine the measure of each $\angle C$ to the nearest degree.

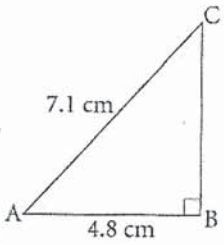
a)



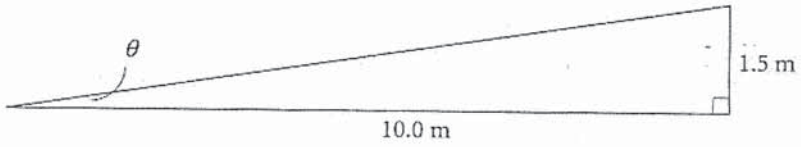
b)



c)

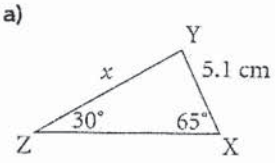


4. A ramp rises vertically 1.5 m for every 10.0 m of horizontal distance. Determine the angle of inclination of the ramp.

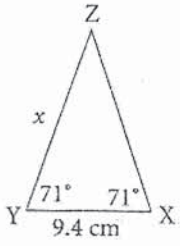


5. Determine the length of each side x.

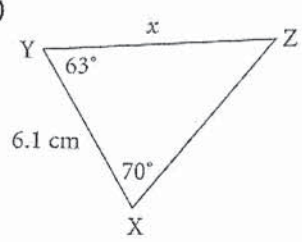
a)



b)

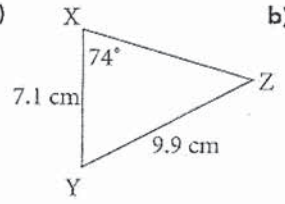


c)

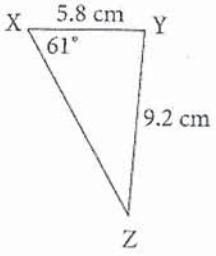


6. Determine the measure of each $\angle Z$ to the nearest degree.

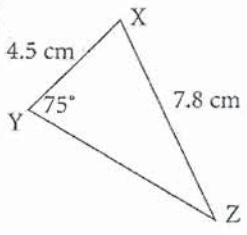
a)



b)



c)



7. At a certain time, the angles of elevation of an airplane, measured from two tracking stations, A and B, 5.8 km apart, are 47° and 53°. Determine the distance between the airplane and tracking station B. (Assume the tracking stations and the airplane are in the same vertical plane.)

8. a) Determine the sine, cosine, and tangent of each $\angle A$.

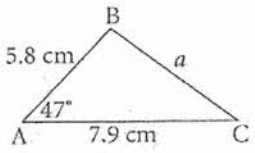
   i) $\angle A = 55°$ ii) $\angle A = 133°$ iii) $\angle A = 79°$

   b) For each $\angle A$ from part a, determine the measure of $\angle B$ so that $\sin B = \sin A$.

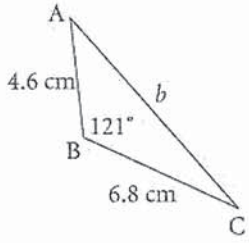
9. Angle M is between 0° and 180° and sin M = 0.4432. Determine the measures of $\angle M$.

10. Determine each indicated measure.

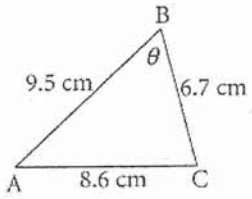
a)



b)



c)



11. Solve each triangle.

   a) $\triangle ABC$ given $\angle B = 61°$, $\angle C = 47°$, and $b = 7.2$ cm

   b) $\triangle PQR$ given $\angle Q = 108°$, $p = 4.7$ cm, and $r = 6.8$ cm

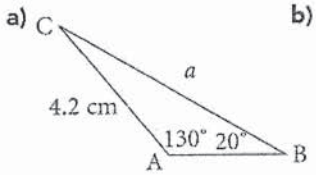
   c) $\triangle XYZ$ given $x = 7.3$ cm, $y = 9.4$ cm, and $z = 5.1$ cm

   d) $\triangle RST$ given $\angle T = 29°$, $r = 6.2$ cm, and $t = 4.8$ cm

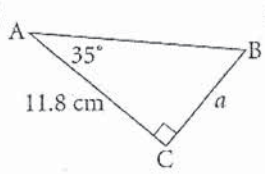
12. From a point on shore, an observer sees two ships at sea. Ship A is 4.9 km from the observer and ship B is 7.3 km from the observer. The angle between the lines of sight to the ships is 67°. Determine the distance between the ships.

13. Determine the length of each side a.

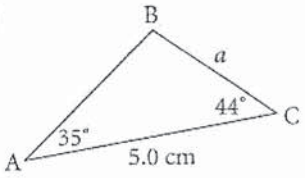
a)



b)



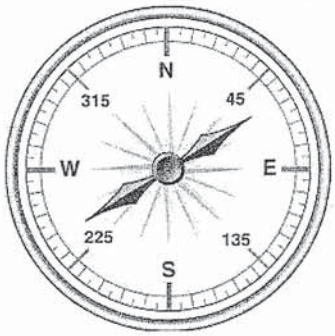
c)



14. A farmer has a triangular field. From one corner of the field, it is 435 m to the second corner and 656 m to the third corner. The angle between the lines of sight to the second and third corners is 49°. Calculate the perimeter of the field to the nearest metre.

15. From the top of a building 70 m high, the angle of depression of a car on a road is 27°. How far is the car from the foot of the building?

16. A sailboat leaves a port on a bearing of 025°. After sailing for 1.5 km, the boat turns and sails on a bearing of 165° for 1.7 km. How far is the boat from port?

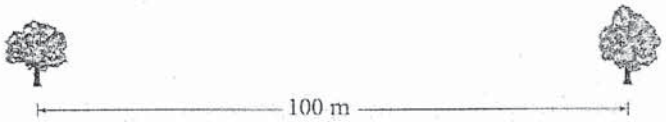


17. Two cabins are in the same vertical plane as a tower, and on opposite sides of the tower. The tower is 118 m high. The angles of depression of the cabins are 10° and 6°. Determine the distance between the cabins.

18. Determine the area of $\triangle DEF$ given $\angle E = 48°$, $d = 15.1$ cm, and $f = 7.9$ cm.

19. From one point, the angle of elevation of the top of a tall building is 5°. From another point, in the same vertical plane 150 m closer to the building, the angle of elevation is 15°. Determine the height of the building.

20. Two trees are 100 m apart. From the midpoint between the trees, Bozena measures the angles of elevation of their tops as 7° and 10°. How tall are the trees?



Answers

1. a) 5.8 cm b) 9.7 cm c) 11.0 cm 2. 52 m 3a) 66° b) 35° c) 43° 4. 8.5° 5a) 9.2 cm b) 14.4 cm c) 7.8 cm
6a) 44° b) 33° c) 34° 7. 4.3 km 8a) i) sin55° = 0.8192, cos55° = 0.5736, tan55° = 1.4281 ii) sin133° = 0.73, cos133° = -0.68, tan133° = -1.07
iii) sin79° = 0.98, cos79° = 0.19, tan79° = 5.14 b) i) 55° or 125° ii) 57° or 133° iii) 79° or 101° 9. 26.31° or 153.69° 10a) 5.8 m
b) 10.0 cm c) 61° 11a) $\angle A = 72°$, a = 7.8 cm, c = 6.0 cm b) $\angle P = 29°$, $\angle R = 43°$, q = 9.4 cm