

**GRADE 7 ONTARIO MATHEMATICS : ESSENTIAL
SKILLS AND REVIEW**

by Mark Burke
Astolot Educational Centre

Introduction : How to use this book

This booklet covers all of the specific curriculum expectations as laid out by the Ministry of Education in Ontario. It is designed to be economical and brief so it is not a comprehensive textbook. Rather it is a companion to whatever textbook a student may have already been provided with. Nonetheless, if students work through the examples (as opposed to just read them !) and then complete the quizzes , they will find they have a sound understanding of the fundamentals which are so important in grade 8 and high school.

Because number sense and algebra are used in most areas, it is especially important that students are able in these core areas in order that they make further progress in mathematics. *In particular, being able to work with order of operations, manipulate equations, and operate with fractions are the key skills* that will enable students to deal with most mathematical problems.

CONTENTS

1.	<u>Number Sense.....</u>	<u>3</u>
2.	<u>Measurement.....</u>	<u>11</u>
3.	<u>Patterns and Algebra.....</u>	<u>17</u>
4.	<u>Geometry and Spatial Sense</u>	<u>21</u>
5.	<u>Probability and Data Management.....</u>	<u>26</u>
	<u>Quiz Solutions</u>	<u>31</u>

1. NUMBER SENSE AND NUMERATION

MULTIPLES AND FACTORS

Multiples are found from the times table of a number ; multiply the number by 0,1, 2 ..etc to generate multiples :

e.g the multiples of 14 are 0, 14, 28, 42, ...etc

Factors of a number are numbers that divide into that number exactly .

e.g the factors of 14 are 1, 2, 7, and 14.

If you group them in pairs you will not miss any. For example the factors of 14 could be written :

$$1 \times 14$$

$$2 \times 7$$

So the factor list for 14 is : 1, 2, 7 14.

PERFECT SQUARES AND SQUARE ROOTS

Perfect squares are numbers that are the result of a number times itself .Examples are 4, 9, 16., 25... because they can be written as 2×2 , 3×3 , 4×4 , 5×5 ...

A square times tables is ;

$$1^2 = 1$$

$$2^2 = 4$$

$$3^2 = 9$$

$$4^2 = 16$$

$$5^2 = 25$$

$$6^2 = 36$$

Square roots are the single numbers that a square number comes from, eg. 2, 3, 4, 5, .

The square root of 81 for example is 9, because $9 \times 9 = 81$.

That is, $\sqrt{81} = 9$

EXPONENTIAL NOTATION AND MEASUREMENT

Area is written in units squared (eg cm^2) because it is two dimension multiplied, ie.

Length x width

e.g A square of dimensions 3cm by 2cm is 6cm^2

Volume is 3 dimensions multiplied by each other so it written as units cubed,

e.g $\text{cm} \times \text{cm} \times \text{cm} = \text{cm}^3$

DIVIDING WHOLE NUMBERS BY FRACTIONS

To divide a number by a fraction, multiply but turn the fraction *upside down*. Remember that fractions multiply as *top x top, bottom by bottom*.

e.g 7 divided by $\frac{2}{3}$ = $7 \times \frac{3}{2}$
 = $\frac{7}{1} \times \frac{3}{2}$
 = $\frac{21}{2}$
 = 10 and $\frac{1}{2}$

COMPARE AND ORDER DECIMALS

Decimals are 'backwards' in the sense that the further you go right from the decimal point, the smaller the number.

0.1 = 1 tenth or $\frac{1}{10}$
 0.01 = 1 hundredth or $\frac{1}{100}$
 0.001 = 1 thousandth or $\frac{1}{1000}$

0.1 is bigger than 0.01, which is bigger than 0.001

This is because 0.1 means $\frac{1}{10}$, $0.01 = \frac{1}{100}$ and $\frac{1}{1000} = 0.001$

You can see 1 decimal place is tenths, 2 d.p is hundredths, and 3d.p = thousandths.

e.g $0.675 = 6$ tenths, 7 hundredths and 5 thousandths
 or $0.6 + 0.07 + 0.005$

Note that $0.2 = 0.20 = 0.200$, so

$\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$

ROUNDING DECIMALS

Rounding decimals is like rounding regular numbers ; look to the right of the number you

want to round to. If the number to the right is 5 or more, round the specified number up.

eg. 0.857 , round to 1 decimal place.

Since we are rounding the first decimal place (the 8) we look at the number to the right (the 5). 5 rounds up, so 0.857 becomes 0.900 Note how all numbers to the right of the 9 are now zero.

eg. 2 :

0.451 rounded to 2 d.p is 0.450 because the 1 will not affect the 5 .

DIVIDING WHOLE NUMBERS BY DECIMALS AND OTHER OPERATIONS

It is often easiest to move the decimal place on both numbers then divide as normal.

eg. 60 divided by 0.05 can be rewritten as 600 / 5 by multiplying both numbers by 100.

When multiplying by decimals, multiply by the integers, then put the decimal place in afterwards.

e.g 5×0.008

= $(5 \times 8) \times 0.001$ (or put 3 decimal places back into the answer)

= $40 \times .001 = 0.040$ (move the decimal 3 places left)

BEDMAS AND ORDER OF OPERATIONS

Brackets first, then exponents, then division and multiplication, then addition and subtraction

e.g $(4 - 2) \times 5^2 - 6 \times 2 = 2 \times 25 - 12$

= $50 - 12$

= 38

The most common example is to simply multiply before adding :

e.g $4 + 8 \times 3 = 4 + 24 = 28$ (NOT $12 \times 3 = 36$)

OPERATIONS WITH FRACTIONS INVOLVING DIFFERENT DENOMINATORS

To add or subtract fractions you need the same denominators (the bottom of the fraction) on each fraction.

To get the same denominators you use common multiples.

For example to add $3/4$ to $5/6$ you would need to make both denominators 12 (12 being the LCM)

$3/4$ becomes $9/12$ because you have to multiply the fraction by 3 to get from 4 to 12.

$5/6$ becomes $10/12$ because you have to multiply the fraction by 2 to get from 6 to 12.

Now they can be added because they are both 'twelfths' : $9/12 + 10/12 = 19/12$

$19/12$ is an improper fraction because the top is bigger than the bottom. Divide 12 into 19 and it goes 1 time remainder 7.

So, $19/12 = 1$ and $7/12$

INTEGERS

Integers are whole numbers that can also be negative ,
e.g -2, -1, 0 , 1 , 2

ADDING AND SUBTRACTING INTEGERS

Adding negative numbers to positive numbers is the same as subtraction.

$$5 + (-6) = -1.$$

It is often useful to think of how a thermometer measures temperature. If the temperature is 5 degrees and drops 6, then it has moved to -1.

Adding only negative numbers is the same as addition except that you put a negative sign in front of the answer. e.g $-3 + -4 = -7$.

FRACTIONS, DECIMALS, PERCENTAGES

A fraction can be converted into a decimal by dividing the denominator into the numerator.

eg. $2/3 = 0.6666...$

Doing this manually requires long division.

Here is an example of long division where $1/7$ is converted into a decimal.

Note how $1/7$ literally means 1 divided by 7.

$$\begin{array}{r}
 0.1428 \\
 7 \overline{) 1.000000} \\
 \underline{1 \times 7 = 7} \\
 30 \\
 \underline{4 \times 7 = 28} \\
 20 \\
 \underline{2 \times 7 = 14} \\
 60 \\
 \underline{8 \times 7 = 56} \\

 \end{array}$$

In the example above, we begin by trying to divide 7 into 1. That will not go, so we move across and try 7 into 10. That goes once, and makes 7. The remainder is 3, and we drop down another zero and now try 7 into 30.....repeat steps

To Convert a Decimal into a Percentage , multiply by 100 or move the decimal point 2 places right .

In this case then $0.1428 \times 100 = 14.28 \%$

Another example : $0.66 = 66 \%$

To Convert Percentages back to Fractions, put the number over 100. Reduce if possible :

e.g $66 \% = 66/100 = 33/50$

Note : fractions should be 'reduced' or simplified by dividing both top and bottom by a common factor.

e.g $12 / 72$ can be divided by 12 : $12/72 = 1/6$

To Convert Decimals to Fractions, put the number over the required number of decimal

places. If there is 1 decimal place, put the number over 10. If there are 2 decimal places, put the number over 100, and so on.

eg. $0.6 = 6/10$ and $0.66 = 66/100$ and $0.666 = 666/1000$

Percentage of a number can be found by multiplying by $n / 100$ where $n =$ the desired percentage.

e.g. 32 % of 68 is $32/100$ multiplied by 68. Just multiply 32 by 68 and then divide by 100 by moving the decimal two places.

Use a calculator, or Multiply using distributive property :

$$\begin{aligned} 32 \times 68 &= (30 + 2) * (60 + 8) \\ &= 30 \times 60 + 30 \times 8 + 2 \times 60 + 2 \times 8 \\ &= 1800 + 240 + 120 + 16 \\ &= 2166 \end{aligned}$$

Move the decimal two places left : 21.66

So 32 % of 68 is 21.66

If the number you are finding a percentage of can fit nicely into 100, then use equivalent fractions .

e.g. to find 34 % of 20, use $34/100 = n/20$

To get from 100 to 20, divide by 5. Do the same to 34 , so $n = 34/5 = 6.8$

RATIOS

Ratios can be treated like fractions.

The ratio 6 : 10 can be simplified as would be $6/10$

So, $6 : 10 = 3 : 5$

RATE is a ratio of different units.

For example if a car moves at speed or a rate of 60km/h, this is the ratio 60km : 1 h

UNIT RATES AND PRICE COMPARISONS

..are calculated by finding the rate of measurement to 1 of the other measurements.

For example, to compare 2 different size juice containers on value for money, find out how, much each costs per 1 ml.

The large juice costs \$4.99 for 700ml, and the medium juice costs \$ 2.99 for 500ml.

For both juices, divide the price by the ml to get the unit rate of cost per ml.

Large : $499 \text{ cents} / 700\text{ml} = 0.71 \text{ c/ml}$

Medium : $299 \text{ cents} / 500\text{ml} = 0.598 \text{ c/ml}$

So the larger juice has a higher unit rate and costs more.

NUMBER SENSE QUIZ (please attempt without a calculator first)

- 1.) Round 0.543857 to 2 decimal places.
- 2.) Which is bigger, 0.9 or 0.11 ?
- 3.) Calculate the HCF and the LCM of 8 and 18.
- 4.) Divide 8 by $\frac{1}{3}$
- 5.) Divide 60 by 0.2
- 6.) Multiply : 76×0.03
- 7.) Add : $\frac{2}{3} + \frac{4}{5}$
- 8.) Multiply ; $\frac{2}{5} \times \frac{6}{7}$
- 9.) Calculate the unit costs of 5 litres of milk at \$7.99, and 3 litres at \$4.99
- 10.) Evaluate : $20 - 4 \times 3 + 6$
- 11.) Add ; $-8 - (-9) =$
- 12.) Convert $\frac{9}{12}$ into a percentage
- 13.) Convert 18 % into a fraction and then simplify
- 14) Simplify the ratio ; 30 : 96
- 15) Solve for the missing proportion ; $\frac{3}{12} = \frac{2}{x}$

16) Calculate 15 % of 28

- Now Check your answers at the back !

2. MEASUREMENT

Measurement Relationships

Different shaped containers may have the same volume.

For example a container with 200cm^3 may be made with a length 10, width 10 and height 2, ($V = 10\text{cm} \times 10\text{cm} \times 2\text{cm} = 200\text{cm}^3$)

OR length 4 , height 5, and width 10 ($V = 4\text{cm} \times 5\text{cm} \times 10\text{cm} = 200\text{cm}^3$)

Metric Conversions

The metric system is based on multiples of 10. A unit is converted to tenths, hundredths, and thousandths. The example below considers metres.

Kilo = 1000 m

hecto = 100m

deka = 10m

deci = 0.1m

centi = 0.01m

milli = 0.001m

A thousand of a unit uses Kilo, e.g a thousand metres is a kilometre. A thousand *th* of a metre is a millimetre.

To convert between units, divide or multiply by tens.

E.g to convert 3.7 km into cm, multiply by 10,000 since there are 4 gaps of tens between the units.

To convert from smaller units to bigger units, *divide since there will be less units.*

e.g to convert 45mm to metres, divide by 10^3 or 1000 : $45\text{mm} = 0.045\text{m}$

Other common conversions

1 kg = 1000g e.g 2.23kg = 2230 g

1 gram = 1000mg eg. 500Mg = 0.5 g

1 litre = 1000 millilitres (ml) e.g 330ml = 0.330 Litres

Conversions for units squared (when converting area measurements)

For converting area measurements, you have to square the conversion factor.

1 metre is 100 cm in one dimension, but $1\text{m}^2 = 100 \times 100 \text{cm}^2 = 10,000 \text{cm}^2$.

To go back to bigger units from smaller units, likewise you must use the conversion factor twice except this time you must divide.

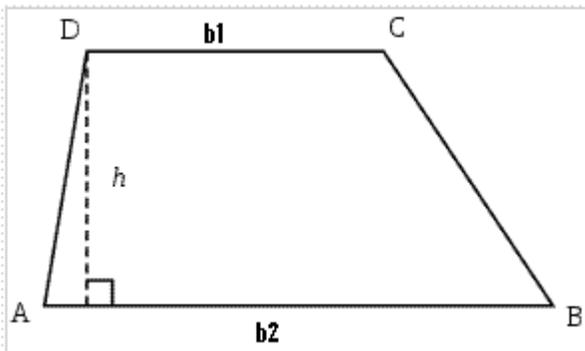
For example ;

$$\begin{aligned} 5000\text{cm}^2 &= 5000/100^2 \\ &= 5000/ (100 \times 100) \\ &= 0.5 \text{m}^2 \end{aligned}$$

Areas of Trapezoids

The formula for Areas of a trapezoid can be thought of as similar to a rectangle except that you are using average length from the two different lengths.

In the example below average lengths b_1 and b_2 using $(b_1 + b_2) / 2$ which is then multiplied by the vertical height 'h' . Note that the slant height is not used.

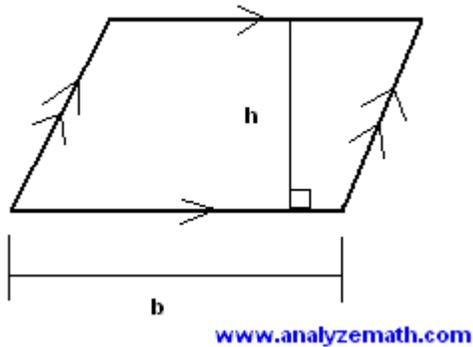


For example in the parallelogram which has vertical height 6cm and two base lengths of 3 and 5 cm, the area is $6 \times 4 \text{cm} = 24 \text{cm}^2$

Area of a Parallelogram

A parallelogram can be thought of as a rectangle pushed over, except that you will still use vertical height multiplied by length.

In fact it may be more useful to think of parallelogram as a trapezoid slotted together with another upside down trapezoid :



In the case above, area is simply ; $A = b \times h$. Again, slant lengths are NOT used in the calculation of area.

Areas of Right Prisms

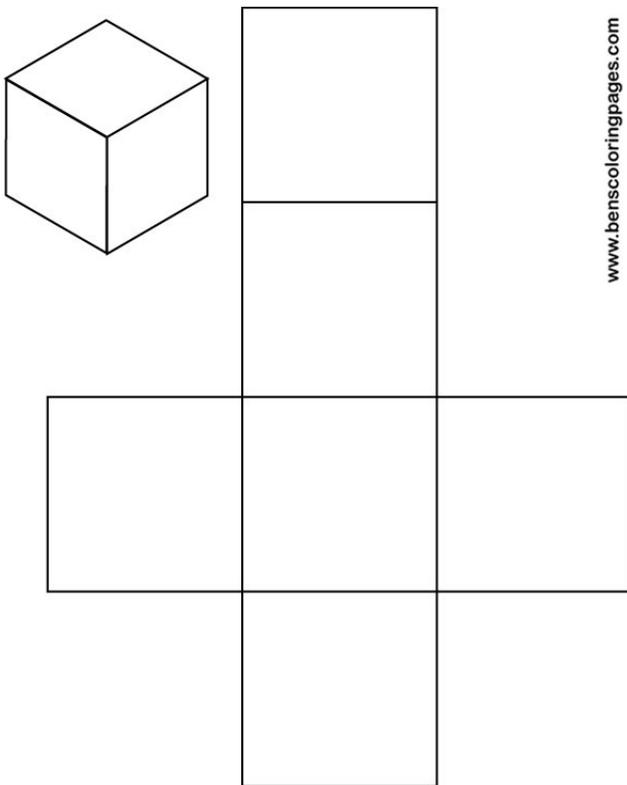
The surface area of 3-D shapes can be found by simply adding the areas of all the faces.

For rectangular prisms you can use the simplified formula :

$$2 [(l \times w) + (h \times w) + (l \times h)]$$

where the area of the top, side and front is found and then doubled because of the opposite sides, eg. the top has the bottom.

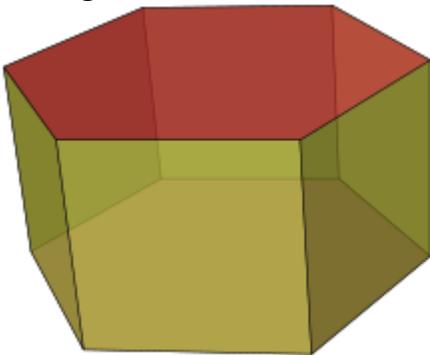
Visually, imagine a box being opened and flattened :



Volume of a prism is three dimensions multiplied.

If a cuboid, simply use $V = \text{length} \times \text{width} \times \text{height}$

A right prism or regular prism is a shape that has 'slices' the same shape all the way through.



In the example above, if the area of the top face was known (say, 20cm^2) then just multiply by the number of 'slices' or height (say 6cm).

In this case then volume = $20\text{cm}^2 \times 6\text{cm} = 120\text{cm}^3$

Unit Conversions and Volume

Note that $1 \text{ cm}^3 = 1\text{ml}$ and this can be used to find the capacity or volume of a shape in litres if you know its dimensions.

e.g an aquarium that has dimensions 20cm by 14 cm by 10cm = 2800cm^3
= 2800 ml
= 2.8 Litres

MEASUREMENT QUIZ (drawing your own diagrams will be useful)

- 1) Convert 423g into kilograms
- 2) Convert 2057ml into litres
- 3) Convert 465cm^2 into m^2
- 4) Calculate the area of a trapezoid that has height 7cm, and lengths 4cm and 6 cm
- 5) Calculate the volume of a triangular prism that has dimensions 5cm x 4cm x 6cm.
Then calculate the volume in litres.
- 6) Calculate the surface area of a cuboid that has dimensions 6cm by 3cm by 2cm.
- 7) Calculate the area of a triangle which has height 70cm and width 40cm.
- 8) Calculate the surface area of a triangular prism which has length 8cm, width 5cm, height 7cm and slant height 9cm (a diagram will help)
- 9) Convert 55 km into metres
- 10) Convert 60m^3 into cm^3

3. PATTERNS AND ALGEBRA

Basic Concepts in Algebra

unknown numbers can be represented by letters known as variables e.g n
variables can be added or 'collected as like terms' e.g $2n + 3n = 5n$

The number in front of a variable is the 'coefficient' and means multiply the variable by that number eg $5n$ means $5 \times n$.

if $n = 3$, then $5n = 5 \times 3 = 15$

REPRESENTING GROWING LINEAR PATTERNS

Most simple linear patterns are represented by looking at the differences between terms.

Suppose you were given a list of numbers and there appeared to be a pattern, such as 5, 8, 11....etc.

Clearly you can see the pattern is that you add 3 to get to the next term.

Mathematics give you a way to predict far into the future. Because there is a difference of 3 this can be modelled with the 3 times tables.

We use $3n$ which means ' $3 \times n$ '.

If you compare the 3 times tables to this sequence, you see this sequence is 2 more, ie the 3 times table with 2 added on.

The 3 times table with 2 added on can be written ; $3n + 2$. where n is the term.

For example ; The 100 term would be : $3 (100) + 2 = 302$.

Some sequences will be times tables you are unfamiliar with, but nonetheless they can be modelled with a simple multiplicative relationship.

For example, consider the sequence 8, -3, 14. ...

The difference between terms is -11, so we can use the '-11' times table.

Now compare the first term 8 to -11.

8 is 19 bigger than -11, so we can use the formula $-11n + 19$.

The 30 term for example would be : $-11 (30) + 19$

$$= - 330 + 19$$

$$= - 311.$$

T-Tables

Sequences can also be represented by tables that show inputs and outputs. Below is a table that represents the sequence generated by ' $3n + 4$ '

Term	Output
1	7
2	10
3	13

Translating statements using algebraic equations

Using x to represent an unknown quantity, and defining other unknowns in terms of x , we can solve complex word problems.

For example, suppose the age of Brett is unknown. But we do know Lisa is 2 years older than him, and Maria is twice as old as Lisa. The total of their ages is 30.

$$\text{Brett} = x$$

$$\text{Lisa} = x + 2$$

$$\text{Maria} = 2(x + 2) = 2x + 4$$

$$\text{Total} : x + (x + 2) + 2x + 4 = 30$$

$$\text{Add like terms and simplify : } 4x + 6 = 30$$

$$4x = 24$$

$$x = 6$$

So, Brett is 6, Lisa is $6 + 2 = 8$, and Maria is $2(6 + 2) = 16$

Solving Linear Equations using the balance model

Our aim here is to shuffle the equation around until we have $x =$

In the above example, we solved an equation using two basic steps ;

- 1) We subtracted the constant (the number on its' own). You can think of this as balancing an equation by subtracting 6 from both sides.

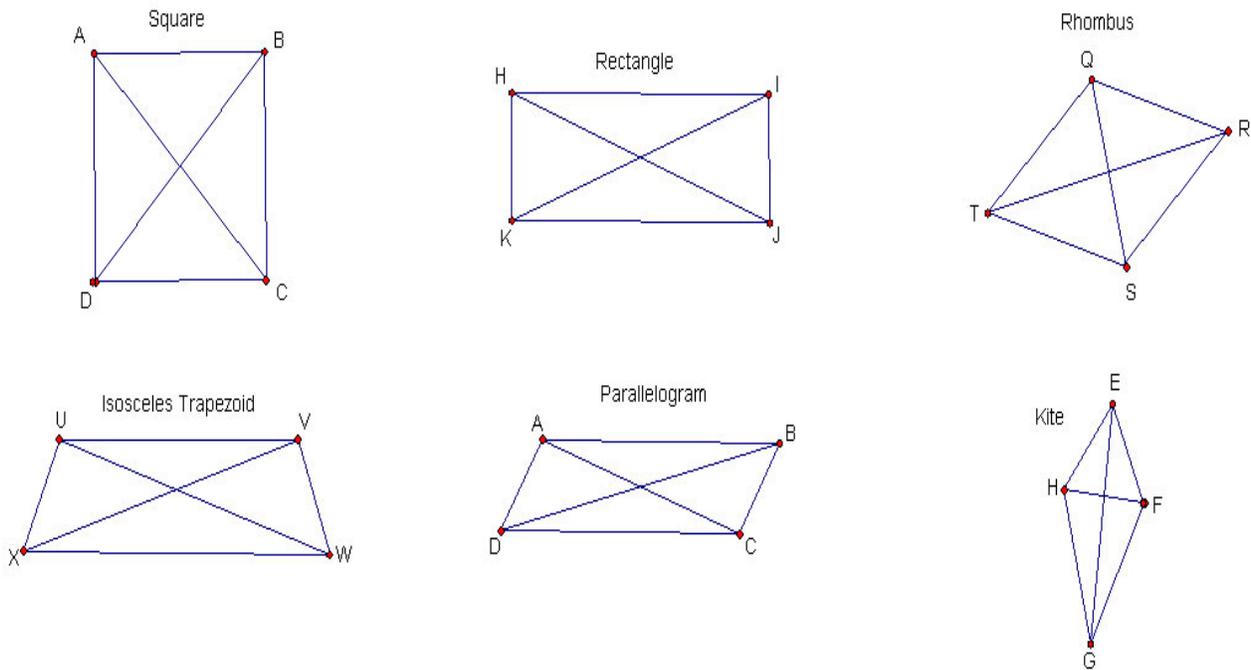
$$4x + 6 - 6 = 30 - 6$$

$$\text{so: } 4x = 24$$

Term	Output
1	17
2	14
3	11

4. GEOMETRY AND SPATIAL SENSE

SORTING QUADRILATERALS BY GEOMETRIC PROPERTIES INVOLVING DIAGONALS



The diagonals of the square and rhombus are congruent and bisect each other at 90° (*perpendicular to each other*).

The diagonals of the rectangle, trapezoid and parallelogram are congruent and bisect too, but *not* at 90° .

The kite is the odd one out in that the diagonals are **NOT** congruent, but they do bisect at 90°

Angles in Polygons

A polygon is a straight sided shape, such as triangle, but not a circle.

Any polygon can be broken into triangles, and this helps you calculate the total interior degrees, as each triangle has 180° inside.

A polygon always has 2 less triangles inside than it's number of sides. For example, a five sided shape can be split into three triangles. Therefore, a pentagon has 3×180 degrees inside.

Total interior degrees of a pentagon = 540° .

The exterior angle of a polygon is 180° minus the interior.

In a regular pentagon, there are a total of 540° and five sides, so each interior angle is 108° .

Each related exterior angle is $180^\circ - 108^\circ = 72^\circ$

(NOT TO SCALE) Total Interior:

$$(n-2) \times 180^\circ = 6 \times 180^\circ = 1080^\circ$$

In a regular polygon of 8 sides, each interior is $\frac{1080^\circ}{8} = 135^\circ$

exterior = $180^\circ - 135^\circ = 45^\circ$

Exterior angles are also directly related to the number of sides of the polygon.

$360^\circ / \text{exterior angle} = \text{the number of sides.}$

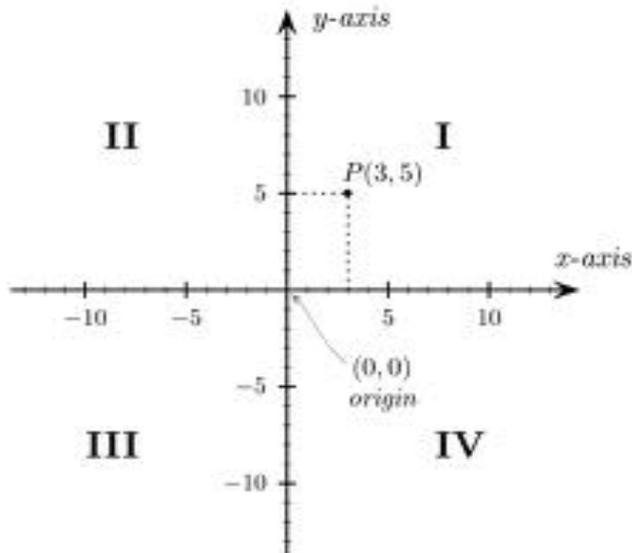
e.g

In a hexagon each exterior angle is $360^\circ / 6 = 60^\circ$

Note how you could also then quickly find the interior 120° .

CARTESIAN GRAPHS AND TRANSFORMATIONS

A Cartesian graph is where points are plotted on x-y axis with 4 quadrants. The point (5, -6) for example is 5 right, 6 down . This would put you in quadrant 3.



Transformations here are when you move a shape on a graph.

A TRANSLATION is moving something up, down or left or right.

Look at the triangle which is defined by the vertices (0,0), (1,0) and (1,2).

If it is moved by the translation (1, 3) this means move all the points up 1, right 3.

A ROTATION is when you spin a shape round, usually around the origin (0, 0). If you rotate by 90° clockwise, your shape will be on it's side, but also move from quadrant 1 to 4.

A rotation of 180° will flip a shape upside down and move into the diagonally opposite quadrant.

A rotation of 270° will move a shape around by 3 quadrants. A shape will be on it' s side again, but the opposite way to a 90° rotation.

A rotation of 360° will return a shape to it's original position.

A REFLECTION is when a shape is reflected across a line and produces a mirror image.

A reflection in the x (horizontal) axis will move a shape into the opposite quadrant.

Notice the mirror effect will appear to flip the shape upside down.

A reflection in the y (vertical) axis will flip a shape from right to left or vice-versa.

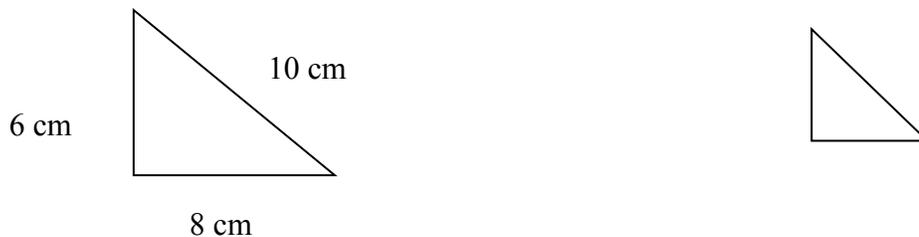
Similar Triangles and Congruency

- *Similar* triangles are triangles that are larger or smaller versions of each other that have the same *angles* and same *ratios* of sides.

- *Congruent* triangles have exactly the same angles *AND* size.

eg. 1:

The triangle that has sides 3, 4 and 5 cm is a smaller *but similar* triangle to that which has sides 6, 8 and 10 cm (*all* sides on the larger triangle are twice as big)



eg. 2 - Triangles which have *all* the same angles inside are similar, regardless of size.

We will now see how we can know with less information, but it is important that *corresponding* sides are similar for the triangle to qualify.

Finding corresponding sides if triangles are similar

If you have 3 out of 4 corresponding sides then you can use ratios to find the missing side. For example if a triangle has height 3cm and width 4cm, and a similar triangle has height 5cm and width 'x' , you can set up a ratio :

$$3/4 = 5/x$$

$(3/4) \times x = 5$ (multiply both sides by x ; x cancels on the right side)

$$x = 5 / (3/4) \quad (\text{divide both sides by } \frac{3}{4} ;$$

$$x = 20/3 \quad (\text{dividing by } \frac{3}{4} \text{ is the same as multiplying by } 4/3)$$

$$x = 6.67 \text{ cm}$$

GEOMETRY AND SPATIAL SENSE QUIZ

1) Construct a rectangle and draw in the lines of symmetry.

2) Calculate the interior and exterior angles in a regular hexagon.

- 3) Plot the point $(-2, -5)$ on a cartesian graph and reflect in the x-axis. Then translate by moving 6 right, 7 up. What is the new co-ordinate ?
- 4) A triangle has height 9cm and width 7cm. A similar triangle has width 21cm. What is its' height ?
- 5) Draw a line AB . Draw in the perpendicular bisector of AB.

5. PROBABILITY AND DATA MANAGEMENT

Discrete data

is data that can only have certain values, such as shoe size. You can only get shoe size $8\frac{1}{2}$ or 9 for example, not shoe size 8.76

Continuous data is data that is in theory unending, because the units used can be further divided. For example you might be 156cm, but your actual height might be 156.75748210....cm etc.

A **Population** is the group that you are examining.

A **Census** is when you get data from *everyone in the population*.

A **Sample** is when you take data from just *some of the population*.

A **Representative Sample** is when your sample, even though it does not get data from everyone, is nonetheless a fair and accurate picture of the population. This is because it tries to get data in a fair way from each 'sub group' that might be in the population. For example, if a school has 25 girls and 15 boys, then a sample might aim to ask survey questions from 5 girls and 3 boys, because this is in the *same proportion*. $25 : 15 = 5:3$

Bias is when a survey can be misleading because it was not done properly. For example a survey that presented the

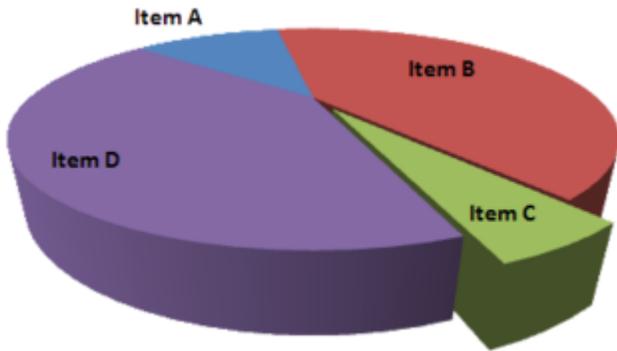
Histograms are bar charts where the bars are next to each other without gaps.

Stem and Leaf diagrams represent data in rows, where the 'trunk' is usually tens and the 'leaves' are the rows which represent units. In the example below, the numbers are 81, 84, 85, 86, 93, 94, 97, 100, 102, 103, 110, 111.

Stem	Leaf
8	1 4 5 6
9	3 4 7
10	0 2 3
11	0 1

Graphs and Charts can be misleading if the axes do not start at zero, because the real gaps between data can be distorted.

Another common trick is to use a 3-D pie chart. The slice facing the front will appear bigger than it really is, because of perspective. (Look at the example below.) Item C is actually only 5 % of the chart but looks a lot more.



Frequency tables are used to collect and organise information. Usually you begin with a tally chart and then a frequency table summarises the data.

Shoe Size	Tally	Frequency
2		2
3		3
4	/	9
5	/	10
6	/	6
7		3
8		2
Total:		35

Measures of Central Tendency

Consider the data set : 5, 8, 8, 9, 3, 1

Mean is when you add all the numbers and divide by how many there are ; in this case :

$$(5+8+8+9+1) / 6 = 5.67 \text{ (2 d.p)}$$

Mode is the most commonly occurring number ; in this case 8

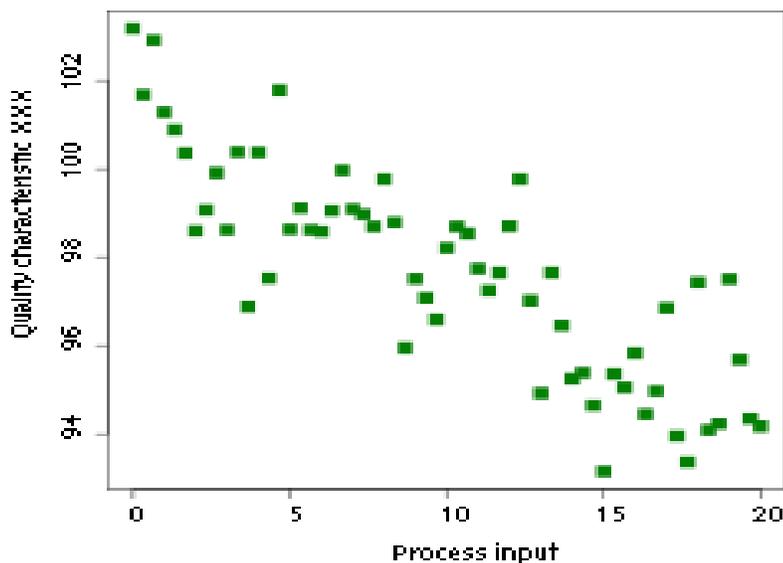
Median is the middle of the data set, when they are in order from smallest to largest. In this case : 1, 3, 5, 8, 8, 9. The middle is between 5 and 8 ; i.e 6.5

Range is the biggest number minus the smallest number. It tells you the spread of your numbers. In this case the range is $9 - 1 = 8$

Outliers are results that are very different to other results. For example, in the data set : 1, 8, 9, 10, 11, 121 is an outlier because it is very far from all the other numbers, which are quite close to each other.

Trends in data refer to a pattern in the data, that usually goes in an upward or downward direction. In the example below there is a downward trend.

Scatterplot for quality characteristic XXX



Probability

Theoretical probability is the probability of a desired outcome divided by the total possibilities.

For example if you throw a dice, the chance of getting a 5 is $1 / 6$ because 6 different things could happen but there is only one 5 on the dice.

Experimental probability is the calculation of probability from a real life experiment or trial .

For example, lets say you flipped a coin 10 times and you got 6 tails and 4 heads.

In theory, the probability of getting a tail is $5/10$ or 0.5 , but in this particular trial the experimental probability is $6/10$ or 0.6

Note that if you repeat an experiment or trial/simulation many times, the closer it will get to the theoretical probability.

In the example above, if we flipped the coin a *hundred* times, we would likely get a probability much closer to the theory.

We might for example get $55/100$ e.g 0.55 or 55% probability

Expectation can be used to predict probabilities. Expectation is measured as the probability of an event occurring times the number of events. For example, the expected number of heads when you flip a coin 100 times is ; $0.5 \times 100 = 50$

Complementary events are events that have to add together to make zero, and are usually the opposites of each other.

For example, if the probability of it raining is 0.3 , then the probability of it NOT raining is $1 - 0.3 = 0.7$ (or $100\% - 30\% = 70\%$)

Independent Events probabilities can be calculated by multiplying probabilities.
e.g the probability of flipping two coins and getting 2 heads is $1/2 \times 1/2 = 1/4$

Organised Lists or Tree Diagrams can show all possible probabilities

eg. all of the possible outcomes of flipping a coin and throwing a dice could be listed as :
H1, H2, H3, H4, H5, H6 and T1, T2, T3, T4, T5, and T6.

Note there are a total of $2 \times 6 = 12$ possibilities.

Any probabilities of these two events can be listed as out of 12.

e.g the probability of getting a Head and Even number is $3/12 = 1/4$ (or 0.25 or 25%)

PROBABILITY AND DATA MANAGEMENT QUIZ

- 1) Calculate the mean, median, mode and range of the data set : 2, 5, 8, 11, 11, 14, 19. If the number 1 was added to the set, would the mean decrease or increase ?
- 2) If the probability of it raining is 0.6 , what is the probability of it NOT raining ?
- 3) What is the theoretical probability of throwing a dice and getting a 3 OR 4 ?
- 4) What is the probability of throwing 2 dice and getting 2 sixes ?
- 5) If a student is late 3 out of 20 days, what is the experimental probability of their lateness ?
- 6) Why is a histogram misleading if the axes do not start at 0 ?

- Now check the answers at the end !

QUIZ SOLUTIONS

NUMBER SENSE QUIZ

- 1) Round 0.543857 to 2 decimal places. = 0.54
- 2) Which is bigger, 0.9 or 0.11 ? $0.9 = 0.90$
- 3) Calculate the HCF and the LCM of 8 and 18.
 Since $8 = 2 \times 2 \times 2$ and $18 = 2 \times 3 \times 3$
 HCF = 2
 LCM = $2 \times 2 \times 2 \times 3 \times 3 = 48$
- 4) Divide 8 by $\frac{1}{3} = 8 \times 3 = 24$
- 5) Divide 60 by 0.2 = $600 / 2 = 300$
- 14.) Multiply : $76 \times 0.03 = 3 \times 76 \times 0.01 = 228 \times 0.01 = 2.28$
- 15.) Add : $\frac{2}{3} + \frac{4}{5} = \frac{10}{15} + \frac{12}{15} = \frac{22}{15} = 1 \text{ and } \frac{7}{15}$
- 16.) Multiply ; $\frac{2}{5} \times \frac{6}{7} = \frac{12}{35}$
- 17.) Calculate the unit costs of 5 litres of milk at \$7.99, and 3 litres at \$4.99
 Cost per litre : $799 / 5 = 159.80$ and $499 / 3 = 166.33$; the second is more expensive
- 18.) Evaluate : $20 - 4 \times 3 + 6 = 20 - 12 + 6 = 14$
- 19.) Add ; $- 8 - (-9) = - 8 + 9 = + 1$
- 20.) Convert $\frac{9}{12}$ into a percentage = $\frac{3}{4} = \frac{75}{100} = 75 \%$
- 21.) Convert 18 % into a fraction and then simplify = $\frac{18}{100} = \frac{9}{50}$
- 14) Simplify the ratio ; $30 : 96 = 10 / 32 = 5/16$

15) Solve for the missing proportion ; $\frac{3}{12} = \frac{2}{x}$; $\frac{1}{4} = \frac{2}{x}$; $x = 8$

16) Calculate 15 % of 28 = $0.15 \times 28 = (280 + 140) \times .01 = 360 \times 0.01 = 3.6$

MEASUREMENT QUIZ

1) Convert 423g into kilograms $423\text{g}/1000 = 0.423\text{kg}$

2) Convert 2057ml into litres = $2057/1000 = 2.057 \text{ L}$

3) Convert 465cm^2 into $\text{m}^2 = 465 / 10000 = 0.0465 \text{ m}^2$

4) Calculate the area of a trapezoid that has height 7cm, and lengths 4cm and 6 cm

$$A = (4+6)/2 \times 7 = 5 \times 7 = 35\text{cm}^2$$

5) Calculate the volume of a triangular prism that has dimensions 5cm x 4cm x 6cm.
Then Calculate the volume in Litres.

$$V = 0.5 (5 \times 4 \times 6) = 60 \text{ cm}^3$$

$$60 \text{ cm}^3 = 60 \text{ ml} = 0.060 \text{ L}$$

6) Calculate the surface area of a cuboid that has dimensions 6cm by 3cm by 2cm.

$$S.A = 2 (6 \times 2 + 6 \times 3 + 3 \times 2) = 2 (12 + 18 + 6) = 72\text{cm}^2$$

7) Calculate the area of a triangle which has height 70cm and width 40cm.

$$A = 0.5 (70 \times 40) = 140 \text{ cm}^2$$

8) Calculate the surface area of a triangular prism which has length 8cm, width 5cm, height 7cm and slant height 9cm (a diagram will help)

9) Convert 55 km into metres = $55\text{m}/1000 = 0.055\text{km}$

10) Convert 60m^3 into $\text{cm}^3 = 60 \times 1000000 = 60,000,000\text{cm}^3$ (60 million cubic centimetres)

PATTERNS AND ALGEBRA QUIZ

1) Find the next two terms, the rule and then the 100th term : 4, 9, 14,
 19, 24, rule is add 5, start at 4. The formula could be '5n - 1'
 $t(100) = 5(100) - 1 = 499$

2) Use the formula rule '6n - 1' to generate the first 5 terms of the sequence
 $6(1) - 1 = 5$
 add 6 : 5, 11, 17, 23, 28,

3) Solve the equations :

i) $3x - 5 = 8$;
 $3x = 8 + 5,$
 $x = 13/3$
 $x = 3 \text{ and } 1/3$

ii) $\frac{3}{4}x + 8 = 19$

$$\left(\frac{3}{4}\right)x = 19 - 8$$

$$\left(\frac{3}{4}\right)x = 11$$

$$3x = 44$$

$$x = 14 \text{ and } 1/4$$

4) Gerald weighs twice as much as Ursula, who weigh 5 kg less than Gurdip. Together they weigh 200kg. What are their individual weights ?

Let Gurdip = x , Ursula = x - 5, and Gerald = 2 (x - 5)

$$x + x - 5 + 2(x - 5) = 200$$

$$2x - 5 + 2x - 10 = 200$$

$$4x - 15 = 200$$

$$4x = 215$$

$$x = 53.25 \text{ kg,}$$

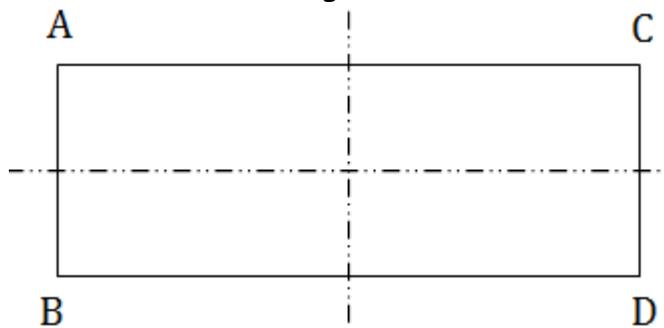
Gurdip is 53.25 kg. Ursula is 48.25 kg, and Gerald is 96.5 kg

5) Complete the table :

Term	Output (gap is -3 so formula is $-3n$. To get from 1 to 17 multiply by 3 and add 14) Rule : $-3t + 14$
1	17
2	14
3	11
4	8

GEOMETRY AND SPATIAL SENSE QUIZ

1) Construct a rectangle and draw in the lines of symmetry.



(Note how unlike in a square, the diagonals are NOT lines of symmetry because the shape would not fold and fit onto itself this way).

2) Calculate the interior and exterior angles in a regular hexagon.

A pentagon has $180 \times (6 - 2) = 720^\circ$. Each interior is $720/6 = 120^\circ$
Each exterior is $180^\circ - 120^\circ = 60^\circ$

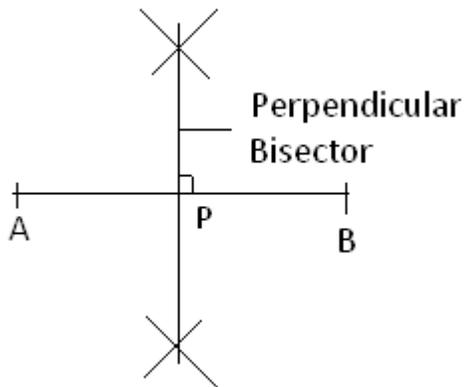
3) Plot the point $(-2, -5)$ on a cartesian graph and reflect in the x-axis. Then translate by moving 6 right, 7 up. What is the new co-ordinate ?

$(-2, 5)$ goes to $(-2, -5)$ then $(4, 2)$

4) A triangle has height 9cm and width 7cm. A similar triangle has width 21cm. What is its' height ?

Ratio of widths is 1 to 3. So height on bigger triangle is $9 \times 3 = 27\text{cm}$

- 5) Draw a line AB. Draw in the perpendicular bisector of AB.



PROBABILITY AND DATA MANAGEMENT QUIZ

- 2) Calculate the mean, median, mode and range of the data set : 2, 5, 8, 11, 11, 14, 19. If the number 1 was added to the set, would the mean decrease or increase ?

Mode is 11.

Median is halfway between 8 and 11 : median is 9.5

mean is $(2 + 5 + 8 + 11 + 11 + 14) / 6 = 51 / 6 = 6.5$

Range is $14 - 2 = 12$

- 2) If the probability of it raining is 0.6 , what is the probability of it NOT raining ?

$$1 - 0.6 = 0.4$$

- 3) What is the theoretical probability of throwing a dice and getting a 3 OR 4 ?

$$2/6 = 1/3$$

- 4) What is the probability of throwing 2 dice and getting 2 sixes ?

$$1/6 \times 1/6 = 1/36$$

- 5) If a student is late 3 out of 20 days, what is the experimental probability of their

lateness ? $3 / 20 = 0.15$

6) Why is a histogram misleading if the axes do not start at 0 ?

Because the difference in heights between the bars is no longer proportional to the start.